Mathématique Pré-Calcul 40S
Unité : Identité Trigonométrique

Nom : ______________________ /38 Date : ______________________

1. Prouve les identités.

a)

$$\frac{2}{1+\cos\theta}+\frac{2}{1-\cos\theta}=4\csc^2\theta \qquad /3$$

$$=\frac{2(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}+\frac{2(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$=\frac{2-\cancel{2\cos\theta}+2+\cancel{2\cos\theta}}{1-\cos^2\theta}$$

$$=\frac{4}{\sin^2\theta} \quad = \quad 4\csc^2\theta$$

✓

b)

$$\frac{\cos^2\theta}{\sin\theta-\sin^2\theta}=1+\csc\theta \qquad /3$$

$\cos^2\theta=1-\sin^2\theta$

$$=\frac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$$

$$=\frac{\cancel{(1-\sin\theta)}(1+\sin\theta)}{\sin\theta\,\cancel{(1-\sin\theta)}}$$

$$=\frac{1+\sin\theta}{\sin\theta}$$

$$=1\frac{\sin\theta}{\sin\theta}+\frac{1}{\sin\theta}$$

$$=\frac{\sin\theta+1}{\sin\theta}$$

$$=\frac{1+\sin\theta}{\sin\theta}$$

✓

c)

$$\frac{\tan\theta\csc^2\theta}{1+\tan^2\theta}=\cot\theta \qquad /3$$

$1+\tan^2\theta=\sec^2\theta$

$$=\frac{\frac{\cancel{\sin\theta}}{\cos\theta}\cdot\frac{1}{\sin^{\cancel{2}}\theta}}{\sec^2\theta}$$

$$=\frac{1}{\cos\theta\sin\theta}\div\frac{1}{\cos^2\theta}$$

$$=\frac{1}{\cancel{\cos\theta}\sin\theta}\cdot\cos^{\cancel{2}}\theta=\frac{\cos\theta}{\sin\theta}=\cot\theta$$

2. Résous de $0 \le \theta \le 2\pi$

$\cot^2\theta + 1 = \csc^2\theta$

a) $2\cot^2\theta - \csc^2\theta = 0$ /3

$2\cot^2\theta - (\cot^2\theta + 1) = 0$

$\cot^2\theta - 1 = 0$

$(\cot\theta - 1)(\cot\theta + 1) = 0$

$\cot\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} + \frac{7\pi}{4}$

b) $\sin^2\theta + 2\cos\theta - 1 = 0$ /3

$(1 - \cos^2\theta) + 2\cos\theta - 1 = 0$

$-\cos^2\theta + 2\cos\theta = 0$

$\cos^2\theta - 2\cos\theta = 0$

$\cos\theta(\cos\theta - 2) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ et $\frac{3\pi}{2}$

$\cos\theta = 2$

aucune solution

c) $2\cos 2\theta + 2 = 2$ /3

solution général en radians

$2(2\cos^2\theta - 1) + 2 = 2$

$4\cos^2\theta - 2 + 2 = 2$

$4\cos^2\theta = 2$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, 5\pi/4, 7\pi/4$

$\theta = \frac{\pi}{4} + \pi n, n \in Z$

$\frac{3\pi}{4} + \pi n, n \in Z$

$\Rightarrow \frac{\pi}{4} + \frac{\pi}{2} n, n \in Z$

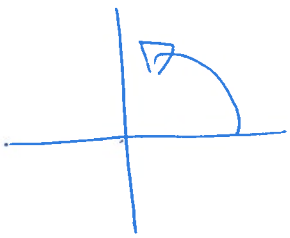
3. Calcule les valeurs exactes de $\sin\theta$ et $\cos\theta$ si $\tan\theta = \frac{7}{3}$ et $0 < \theta < \frac{\pi}{2}$ /2

y/x

$7^2 + 3^2 = r^2$

$\sqrt{49+9} = \sqrt{r^2}$

$\sqrt{58} = r$



$\sin\theta = \frac{7}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$

$\cos\theta = \frac{3}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$

4. Vérifie l'expression trigonométrique pour $\frac{\pi}{6}$. /2

$$\frac{\sin^2\theta}{\cos\theta - \cos^2\theta} = 1 + \sec\theta$$

$$\frac{\sin^2\frac{\pi}{6}}{\cos\frac{\pi}{6} - \cos^2\frac{\pi}{6}} = 1 + \sec\frac{\pi}{6}$$

$$\frac{\left(\frac{1}{2}\right)^2}{\frac{\sqrt{3}\cdot 2}{2\cdot 2} - \left(\frac{\sqrt{3}}{2}\right)^2} = 1 + \frac{2}{\sqrt{3}}$$

$$\frac{1}{4} \div \left(\frac{2\sqrt{3}-3}{4}\right)$$

$$\frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{2\sqrt{3}-3}$$

$$\frac{1}{2\sqrt{3}-3} \cdot \frac{2\sqrt{3}+3}{2\sqrt{3}+3}$$

$$= \frac{2\sqrt{3}+3}{(2\sqrt{3})^2 - 9}$$

$$= \frac{2\sqrt{3}+3}{3}$$

$$= \frac{\sqrt{3}+2}{\sqrt{3}}$$

$$= \frac{(\sqrt{3}+2)\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$= \frac{2\sqrt{3}+3}{3}$$

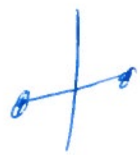
✓

5. a) Détermine les valeurs non permises $\frac{cosec\theta - sin\theta}{sec\theta - cos\theta}$ /2

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta \neq 0$ $\sin\theta \neq 0$

$\theta = \frac{\pi}{2} + \pi n, n \in Z$



$\theta = \pi n, n \in Z$

b) Simplifie l'expression. /2

$$\frac{\frac{1}{\sin\theta} - \sin\theta \cdot \frac{\sin\theta}{\sin\theta}}{\frac{1}{\cos\theta} - \cos\theta \cdot \frac{\cos\theta}{\cos\theta}} = \frac{\frac{1-\sin^2\theta}{\sin\theta}}{\frac{1-\cos^2\theta}{\cos\theta}} = \frac{\cos^2\theta}{\sin\theta} \div \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\cos\theta}{\sin^2\theta}$$

$$= \frac{\cos^3\theta}{\sin^3\theta} = \cot^3\theta$$

6. Détermine les valeurs exactes. /5

a) sin (75°) $\sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b) $\frac{\tan 115^0 + \tan 95^0}{1 - \tan 115^0 \tan 95^0}$

$\tan(115^\circ + 95^\circ) = \tan 210^\circ$

$= \frac{1}{\sqrt{3}}$

$\sin(\alpha-\beta)$

7. Détermine la valeur exacte de $\sin\left(-\frac{11\pi}{12}\right)$ $\sin\left(-\frac{8\pi}{12}-\frac{3\pi}{12}\right)$ /3

$$\sin\left(-\frac{2\pi}{3}-\frac{\pi}{4}\right) = \sin\left(-\frac{2\pi}{3}\right)\cos\frac{\pi}{4} - \cos\left(-\frac{2\pi}{3}\right)\sin\frac{\pi}{4}$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\sin\left(-\frac{11\pi}{12}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$$

Q IV

8. Si $\sin A = -\frac{3}{5}$ et $\cos B = \frac{5}{13}$, A et B sont dans le même quadrant calcule : /4

$\cos(A-B)$ $= \cos A \cos B + \sin A \sin B$

$\cos A = \frac{4}{5}$

$1-\left(-\frac{3}{5}\right)^2 = \cos^2 A$

$\pm\sqrt{\frac{25-9}{25}} = \sqrt{\cos^2 A}$

$+\frac{4}{5} = \cos A$

$\sin B = -\frac{12}{13}$

$1-\left(\frac{5}{13}\right)^2 = \sin^2 B$

$\pm\sqrt{\frac{169-25}{169}} = \sqrt{\sin^2 B}$

$-\frac{12}{13} = \sin B$

$\cos(A-B) = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right)$

$\cos(A-B) = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$