

Perm/Comb, binôme

1. 7 termes \Rightarrow (n+1) alors terme au centre est t_3

$$t_3 = {}_6C_2 \left(\frac{x}{2} \right)^2 (2y)^2$$

$$t_3 = 15 \left(\frac{x^2}{16} \right) (4y^2)$$

$$t_3 = \frac{60x^2y^2}{16}$$

$$t_3 = \frac{15x^2y^2}{4}$$

2. a) $\frac{6!}{2!} = 360$

b) $\frac{2 \cdot 5!}{2!} = 120$

3. ${}_{13}C_4 = 715$

4. ${}_{5P} 1, 4, 6, 4, 1$

1	5	10	10	5	1	6 ^e
1	6	15	20	15	6	7 ^e

5. $(x-3)^5$

$$t_2 = {}_5C_1 (x)^4 (-3)^1 = -15x^4$$

$$t_3 = {}_5C_2 (x)^3 (-3)^2 = 10x^3 \cdot 9 = 90x^3$$

$$t_6 = {}_5C_5 (x)^0 (-3)^5 = 1 \cdot 1 \cdot -243 = -243$$

$$t_1 = {}_5C_0 (x)^5 (-3)^0 = x^5$$

aucun

6. $x^2y^3 \rightarrow (x)^{5-3} (y)^3$
 $k=3$

$${}_5C_4 = 5$$

$$7. \begin{matrix} n=7 \\ r=5 \end{matrix}$$

$${}^n C_r$$

$${}^n P_r$$

$${}^7 C_5 = 21$$

$${}^7 P_5 = 2520$$

8. (D) tu ne peux pas choisir plus d'éléments qu'un total (pas choisir 6 d'un total de 5.)

$$9. \frac{n!}{(n-d)!} = 90$$

$$\frac{n(n-1)(n-2)\dots}{(n-2)!} = 90$$

$$n^2 - n - 90 = 0 \quad \text{dois le montrer}$$

$$(n-10)(n+9) = 0$$

$n=10$ $n=9$ racine étrangère

$$10. (4! 4! 2! 5!) \cdot 4! = 3317760$$

$$11. x^{27} y^5 = (x^{38-k}) y^k + 6$$

$$x y^5 = y^k$$

$$k=5$$

(VPR)

$$(x^3)^{8-5} y^5 = x^{27} y^5$$

Fct Rationnelle / Opération / Fct Polynomial

1. $g(g(-2x+1))$

a) $g(g(x)) = -2(-2x+1)+1$
 $= 4x-2+1$
 $= 4x-1$

b) $g(f(x)) = g(x^2-3x)$
 $= -2(x^2-3x)+1$
 $= -2x^2+6x+1$

c) $f(g(1))$

$g(1) = -2(1)+1 = -1$
 $f(-1) = (-1)^2-3(-1)$
 $= 4$

d) $f(1) = -2$

$g(2) = -3$

e) $f(2)$
 $= -2$

$g(2) = -3$

$\left(\frac{f}{g}\right)(2) = \frac{-2}{-3}$

2. $\frac{f(x)}{g(x)} = \frac{3x+1}{x-2}$

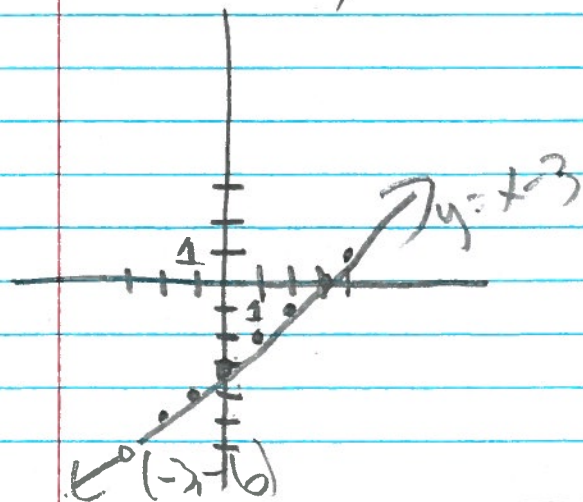
asy vert $x=2$

asy hor $y=3$

b) Dom: $\{x \in \mathbb{R} \mid x \neq 2\}$ Image $\{y \in \mathbb{R} \mid y \neq 3\}$

3. a) $\frac{(x+3)/(x-3)}{x+3} = \frac{f(x)}{g(x)} = x-3$

pt disc. $(-3, -6)$



domaine $\{x \in \mathbb{R} \mid x \neq -3\}$

image $\{y \in \mathbb{R} \mid y \neq -6\}$

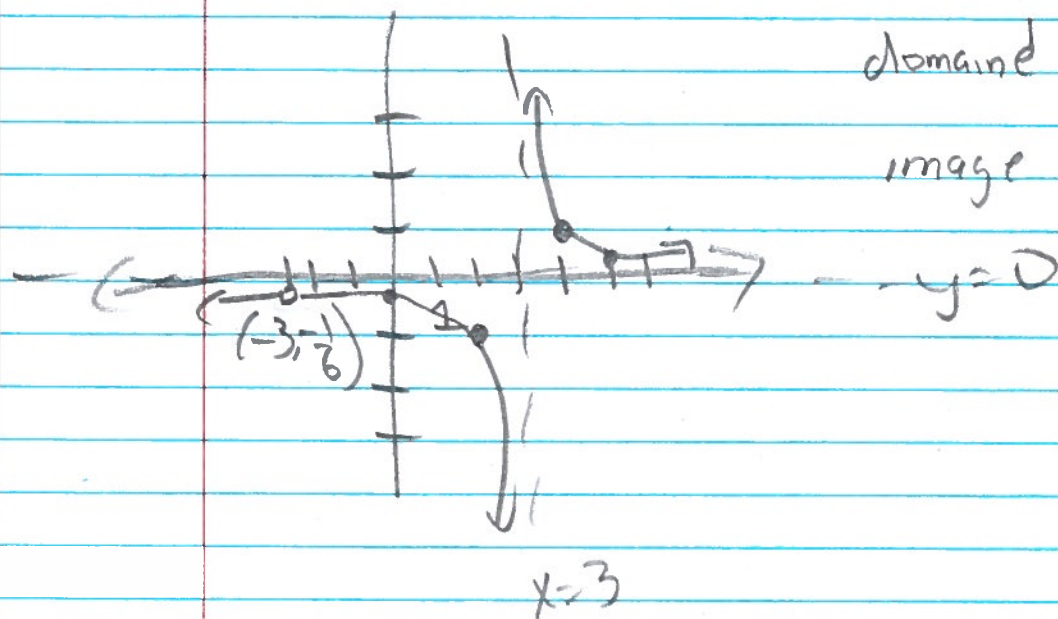
$$b) \frac{g(x)}{f(x)} = \frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3}$$

asy vert $x=3$
asy hor. $y=0$

pt disc. $(-3, -\frac{1}{6})$

domaine $\{x \in \mathbb{R} \mid x \neq -3; x \neq 3\}$

image $\{y \in \mathbb{R} \mid y \neq -\frac{1}{6}; y \neq 0\}$



ord. $y = -\frac{1}{3}$

4, a) $f(g(x)) = 10$

b) $f(f(1)) = 2$

c) $g(x) = 0 \quad x = 3 \quad x = -3$

d) $g(-2) = -4$

$f(-1) = 4$

5.

$g(-2) / f(-1) = \frac{-4}{4} = -1$

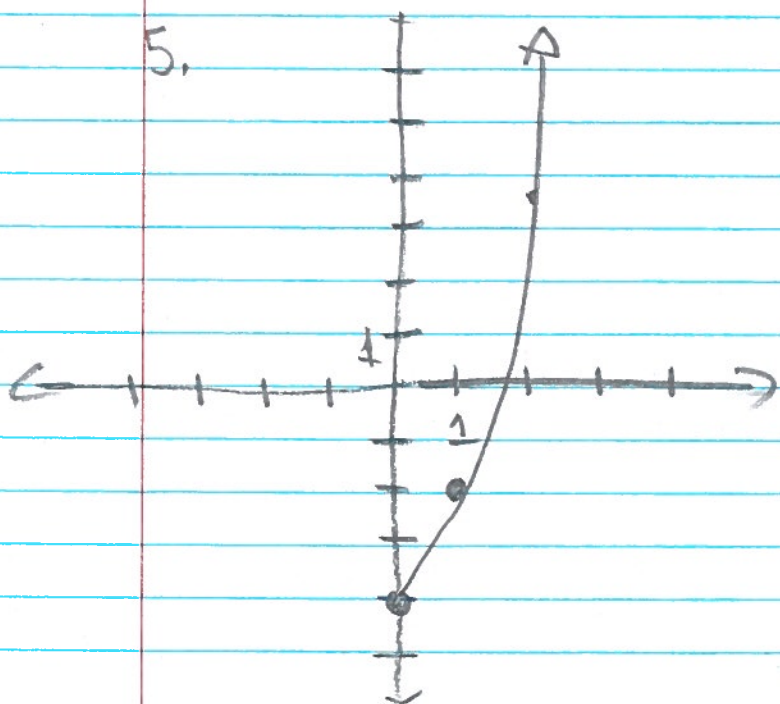
6. $x=3$

$f(3) = -8(3)^2 - 27(3) + 4(3)^3 + 45$

$= -72 - 81 + 108 + 45$

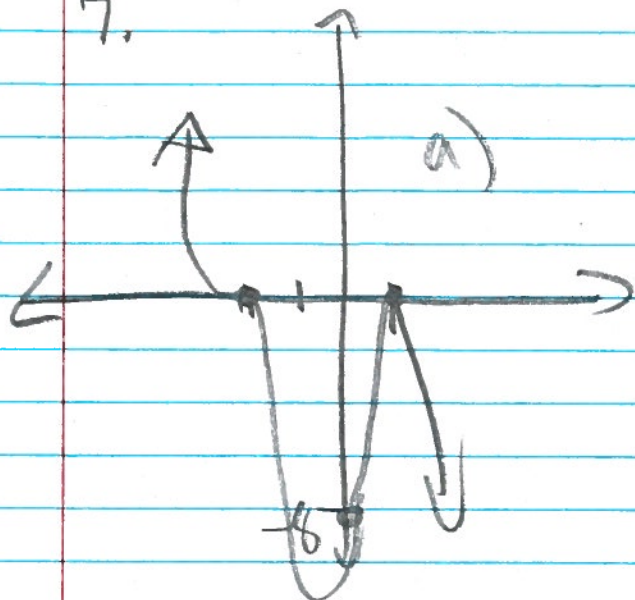
$= 0$

oui il est un facteur



Fct Parabolle etc..

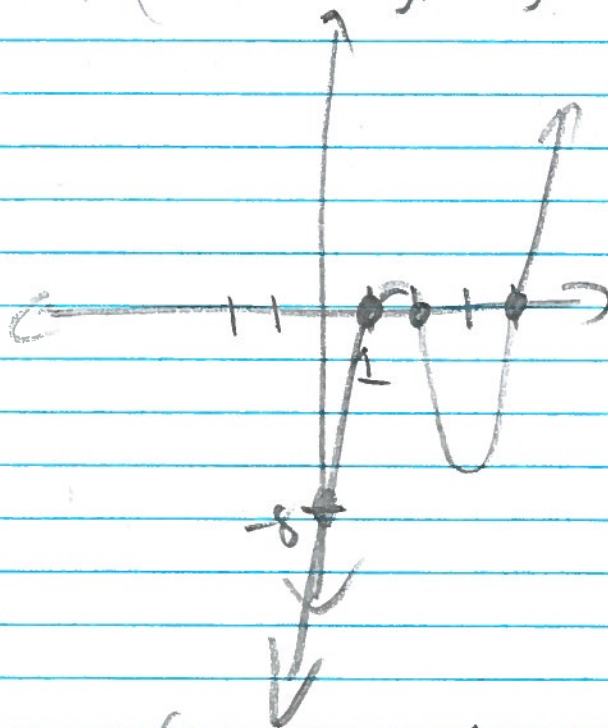
7.



$$\begin{array}{r|rrrr}
 b) & 2 & 1 & -7 & 14 & -18 \\
 + & & 1 & & & \\
 \hline
 x & & 1 & -5 & 4 & 0 \\
 \hline
 \end{array}$$

$$(x-2)(x^2-5x+4)$$

$$f(x) = (x-2)(x-4)(x-1)$$



8. $-x^3 + 35x^2 + 200x$

$$\rightarrow (x^2 - 35x - 200)$$

a) $-x(x-40)(x+5)$

longueur = x ou $x+5$

largeur = x ou $x+5$

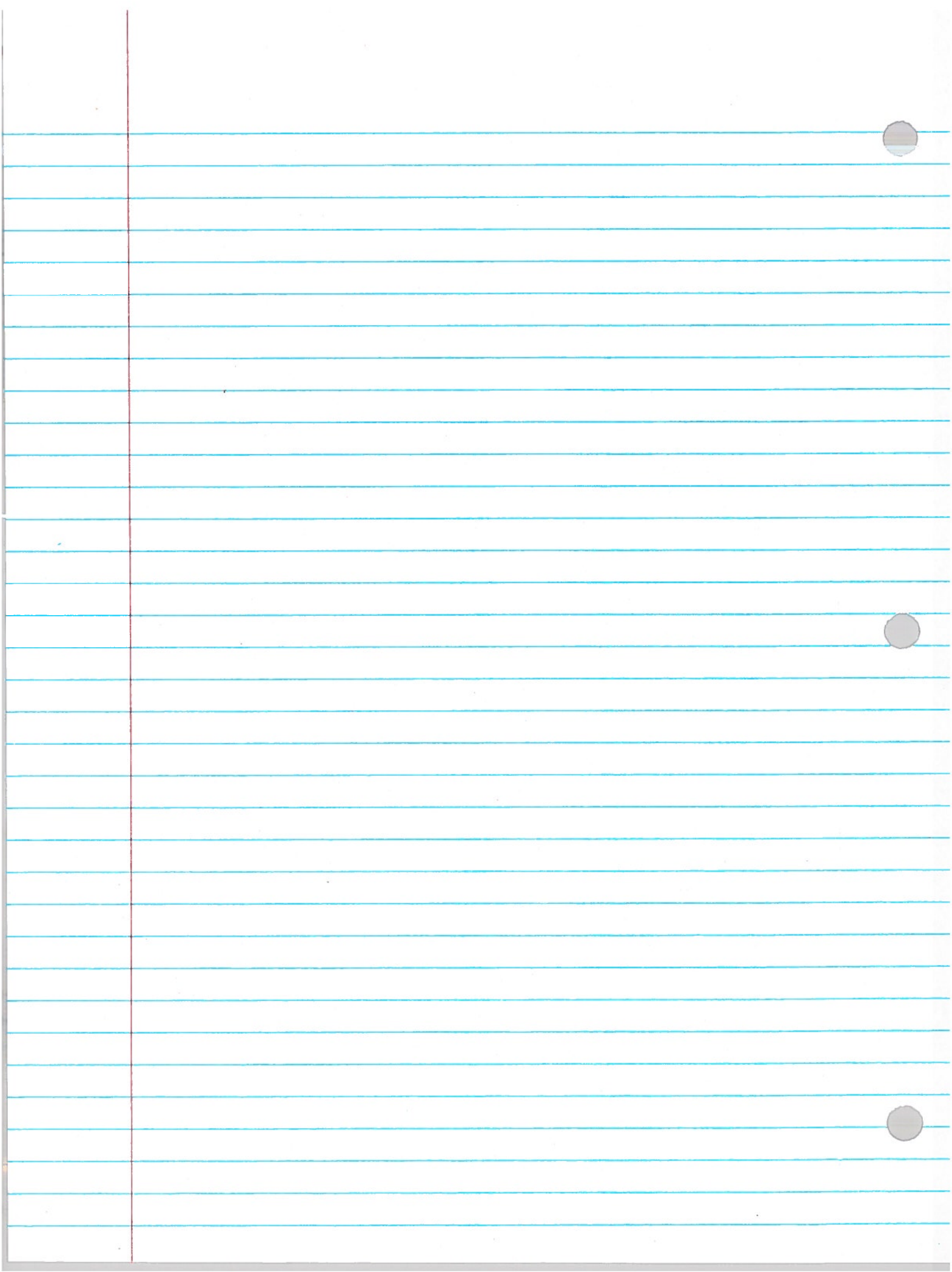
$$\begin{array}{r|rrrr}
 (40-x) \Rightarrow (x=40) & 40 & -1 & 35 & 200 & 0 \\
 + & & 1 & & & \\
 \hline
 & & 1 & -5 & 0 & \\
 \hline
 \end{array}$$

$$(x-40)(-x^2-5x)$$

$$(x-40) - x(x+5)$$

$$-x(x-40)(x+5) \text{ ou } x(40-x)(x+5)$$

b) Vol Max. $(5)(35)(40) = 1750 \text{ cm}^3$



Trig

$$\begin{aligned}
 1. \quad \frac{2 - 2\cos 2\theta}{2\sin 2\theta} &= \frac{\sec^2 \theta - 1}{\tan \theta} \quad \text{or} \quad \frac{1 - \cos^2 \theta}{\cos^2 \theta} \\
 &= \frac{2(1 - \cos 2\theta)}{4\sin \theta \cos \theta} = \frac{\tan^2 \theta}{\tan \theta} = \frac{1 - \cos^2 \theta}{\cos^2 \theta} \cdot \frac{\cos \theta}{\sin \theta} \\
 &= \frac{2(1 - (1 - 2\sin^2 \theta))}{4\sin \theta \cos \theta} = \frac{\sin^2 \theta \cdot \cos \theta}{\cos^2 \theta \cdot \sin \theta} \\
 &= \frac{2(2\sin^2 \theta)}{4\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta \\
 &= \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \checkmark
 \end{aligned}$$

$$2. \quad \cos^2 \alpha = 1 - \left(\frac{2}{3}\right)^2$$

$$\sqrt{\cos^2 \alpha} = \pm \sqrt{\frac{9-4}{9}}$$

$$\cos \alpha = \frac{-\sqrt{5}}{3}$$

$$\sin^2 \beta = 1 - \left(\frac{3}{7}\right)^2$$

$$\sqrt{\sin^2 \beta} = \pm \sqrt{\frac{49-9}{49}}$$

$$\sin \beta = \frac{-\sqrt{40}}{7} = \frac{-2\sqrt{10}}{7}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{-\sqrt{5}}{3} \cdot \frac{3}{7} + \frac{-2}{3} \cdot \frac{-2\sqrt{10}}{7}$$

$$= \frac{-3\sqrt{5}}{21} + \frac{4\sqrt{10}}{21}$$

$$\cos(\alpha - \beta) = \frac{4\sqrt{10} - 3\sqrt{5}}{21}$$

$$\sec(\alpha - \beta) = \frac{21}{4\sqrt{10} - 3\sqrt{5}}$$

$$3. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$1 + \sin \theta \neq 0$$

$$\sin \theta \neq -1$$

$$\sin \theta \neq 0$$

$$\theta = \pi n, n \in \mathbb{Z}$$

$$\theta = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$\cos 5x \sin 2x - \sin 5x \cos 2x$$

$$4. \sin 2x \cos 5x - \cos 2x \sin 5x = \sin(\alpha - \beta)$$

$$\sin(2x - 5x) = \sin(-3x)$$

$$5. \sin(x + \frac{\pi}{2}) = 2(1 - \sin^2 x)$$

$$-\frac{3\pi}{2} < x < \frac{\pi}{2}$$

$$a) \sin(x + \frac{\pi}{2}) = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2}$$

$$\sin x = -1$$

$$x = \frac{\pi}{6}$$

$$x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$x = \frac{7\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$x = \frac{\pi}{6}$$



Trig

$$5. b) 2\cos^2 x - 1 + 5\cos x - 4\cos^2 x + 4 = 0$$

$$-2\cos^2 x + 5\cos x + 3 = 0$$

$$2\cos^2 x - 5\cos x - 3 = 0$$

$$(2\cos x + 1)(\cos x - 3) = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = 3 \quad \text{no real solution}$$

$$x = \frac{\pi}{3}$$

$$x = \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

$$x = \frac{4\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

$$6. \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\sin\frac{\pi}{4} \cos\frac{\pi}{3} + \cos\frac{\pi}{4} \sin\frac{\pi}{3}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$7. 5(2\sin 4x \cos 4x) = 5 \sin 2A$$

$$5 \sin 2 \cdot 4x = 5 \sin 8x \quad \textcircled{D}$$

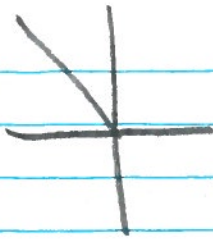
$$\begin{aligned}
 8. \quad -\sec 2\theta &= \frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} \\
 &= \frac{\frac{1}{\cos 2\theta}}{\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta}} \\
 &= \frac{\frac{1}{\cos 2\theta}}{\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta}}
 \end{aligned}$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{\cos \theta \sin \theta}$$

$$\begin{aligned}
 &= \frac{1}{\cos \theta \sin \theta} \cdot \frac{\cos \theta \sin \theta}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{1}{(\cos^2 \theta - \sin^2 \theta)}
 \end{aligned}$$

$$= \frac{1}{\cos 2\theta}$$

$$= -\sec 2\theta$$

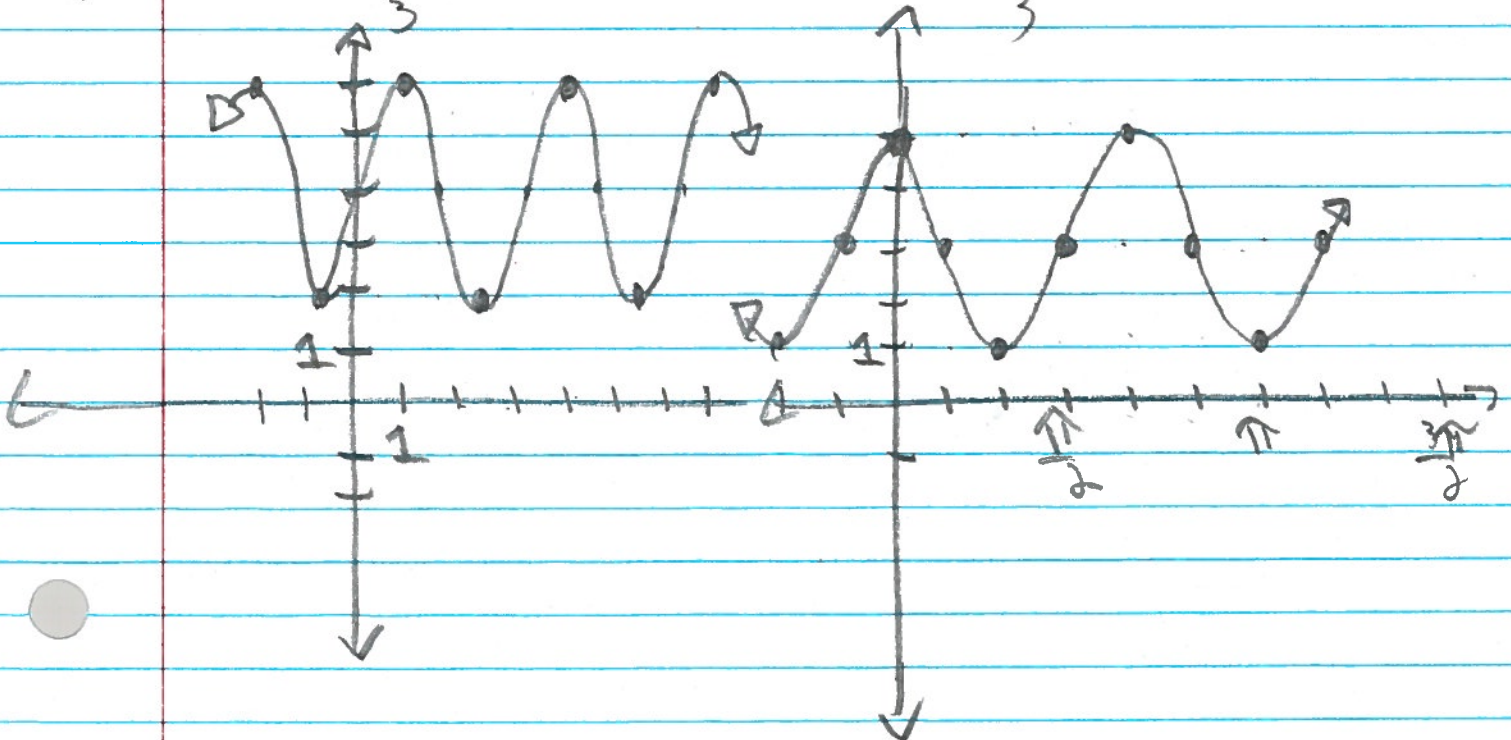
9.  $\cos \theta = -\frac{1}{2}$ $\theta = \frac{\pi}{3}$
 $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{2\pi}{3}$

10. $\cos^2 \theta + \sin^2 \theta = 1$ $\sec \theta = -\frac{5}{3}$ $\cot \theta = \frac{3}{4}$
 $\sin^2 \theta = 1 - \left(-\frac{3}{5}\right)^2$
 $\sin \theta = \pm \frac{4}{5}$
 $\sin \theta = -\frac{4}{5}$ $\csc \theta = -\frac{5}{4}$
 $\tan \theta = \frac{4}{3}$

11. a) $y = 2 \cos \frac{2\pi}{3} (\theta - 1) + 4$ b) $y = 2 \sin 3 \left(x - \frac{\pi}{2}\right) + 3$

max = 6
 min = 2
 periode = $2\pi \div \frac{2\pi}{3} = 3$

max = 5
 min = 1
 per = $\frac{2\pi}{3}$



12. $a = 8$
 période = 10

1) $y = 8 \sin \frac{\pi}{5}(\theta) - 6$

a) $b = \frac{2\pi}{10} = \frac{\pi}{5}$
 $d = -6$

ou $y = -8 \sin \frac{\pi}{5}(\theta - 5) - 6$

b) $a = 2$

période = $\frac{2\pi}{3}$

$y = 2 \cos 3(\theta) - 1$

$b = \frac{2\pi}{3} = \frac{2\pi}{3}$

$y = -2 \cos 3(\theta - \frac{\pi}{3}) - 1$

$b = 3$

$y = 2 \sin 3(\theta - \frac{\pi}{6}) - 1$

$d = -1$

$y = -2 \sin 3(\theta - 1) - 1$

13. $b = \frac{1}{2}$

période = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

14. $\frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$

$\rightarrow \frac{7\pi}{4} + \frac{8\pi}{4}n, n \in \mathbb{Z}$

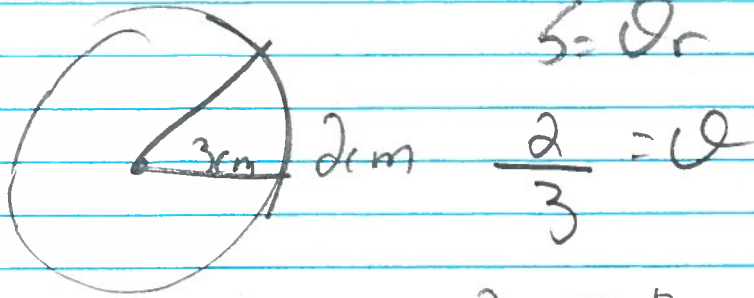
$\frac{7\pi}{4} - \frac{8\pi}{4} = -\frac{\pi}{4}$ ✓

$-\frac{\pi}{4} - \frac{8\pi}{4} = -\frac{9\pi}{4}$ ✓

$\frac{7\pi}{4} + \frac{8\pi}{4} = \frac{15\pi}{4}$ ✓

Ⓐ $-\frac{7\pi}{4}$ Non

15.



$$s = \theta r$$

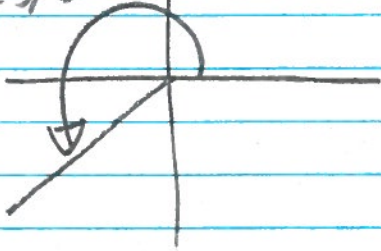
$$\frac{2}{3} = \theta$$

$$\frac{2}{3} \cdot \frac{180^\circ}{\pi} = \theta \quad \theta = 38,197^\circ$$

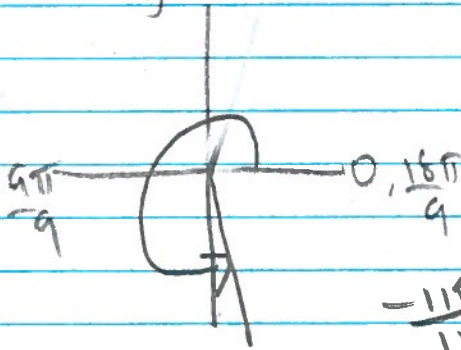
16.

$$\theta = 3,6$$

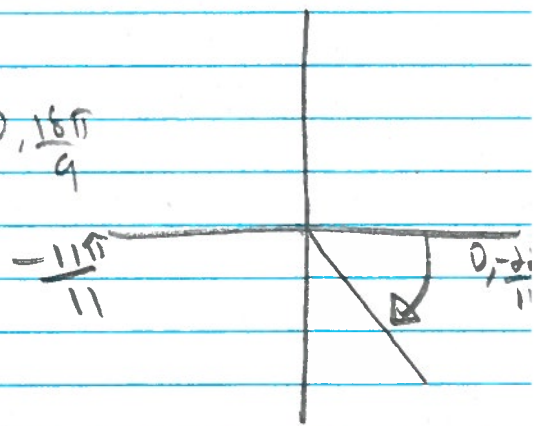
a)



b) $\alpha \frac{14\pi}{9}$



c) $-\frac{3\pi}{11}$



17. $\cos \frac{3\pi}{4} \sin \frac{7\pi}{4} - \sec \frac{5\pi}{3} \tan \frac{7\pi}{4}$

a) $-\frac{\sqrt{2}}{2} \cdot -\frac{\sqrt{2}}{2} - 2 \cdot 1$

$$= \frac{2}{2} - 2$$

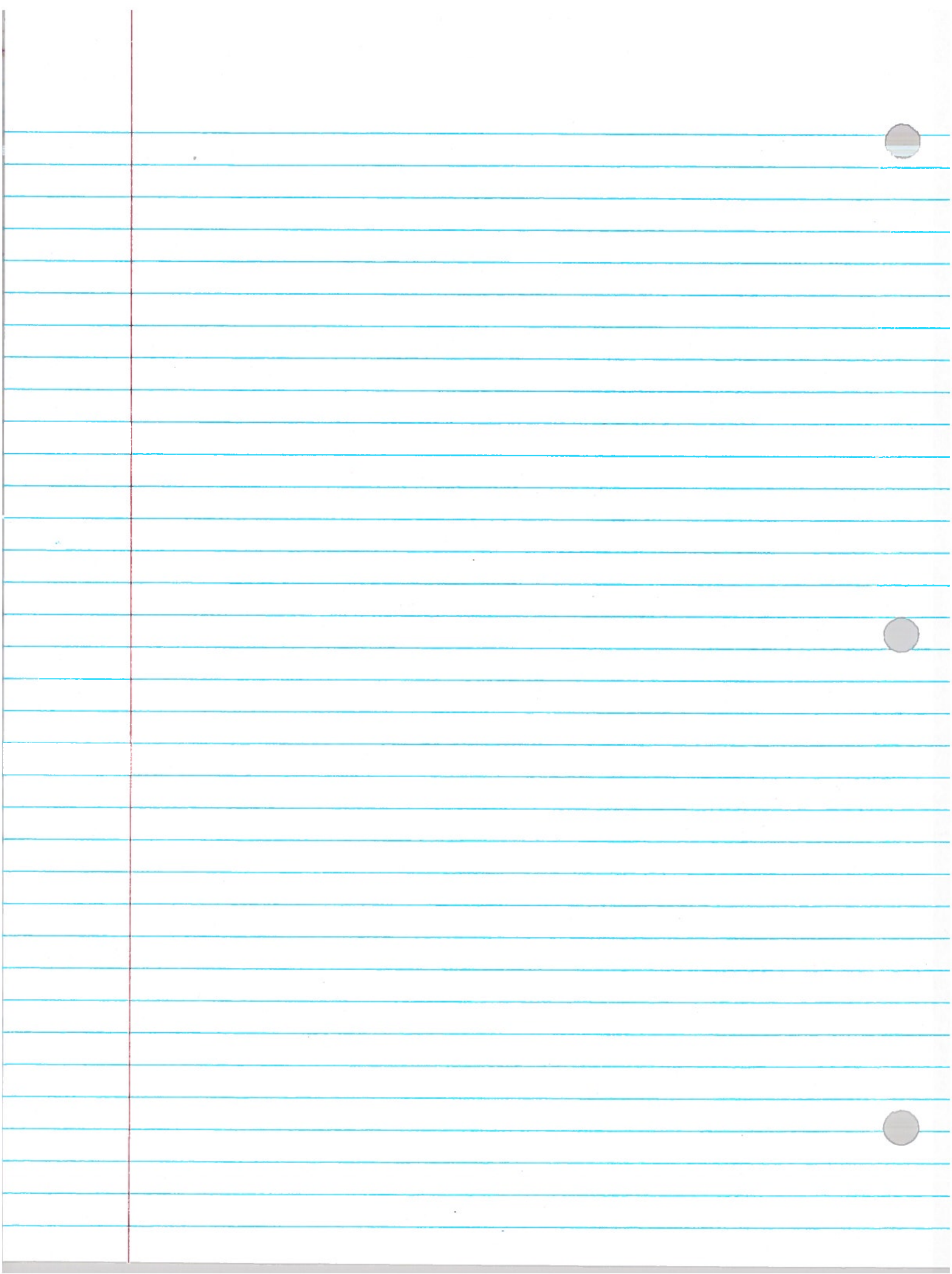
$$= \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$$

b) $\cos^2 \frac{5\pi}{12} - \sin^2 \frac{5\pi}{12}$

$$\cos 2\theta$$

$$\cos 2\left(\frac{5\pi}{12}\right)$$

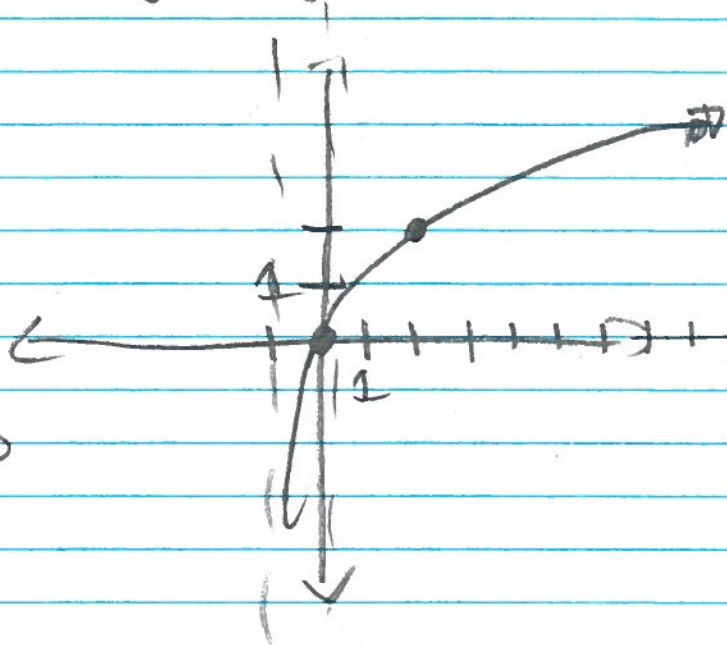
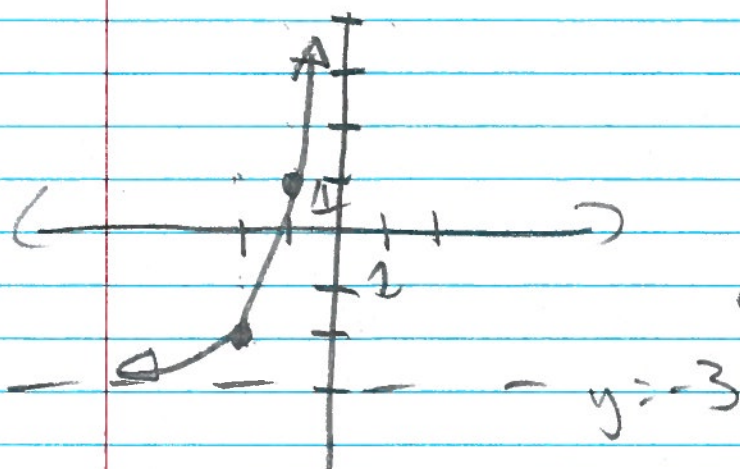
$$\cos \frac{10\pi}{12} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$



Fonctions Exponentielles et log

1. a) $y = 4^{x+2} - 3$

b) $y = 2 \log_3(x+1)$



$D : \{x \in \mathbb{R}\}$

$I : \{y \in \mathbb{R} \mid y > -3\}$

$x = -1$

a) $(3^2)^{2x+1} = \frac{3^{3/4}}{3^4}$

b) $\log_3 \frac{(x+5)(x-1)}{x+1} = 2$

$3^{4x+2} = 3^{3/4} \cdot 3^{-4}$

$3^2 = \frac{(x+5)(x-1)}{x+1}$

$3^{4x+2} = 3^{\frac{3}{4} + (-4)}$

$9x+9 = x^2 + 4x - 5$

$4x+2 = \frac{3}{4} - \frac{16}{4}$

$0 = x^2 - 5x - 14$

$4x+2 = \frac{-13-2}{4} \quad x = \frac{-21}{16}$

$0 = (x-7)(x+2)$

$x = 7$

~~$x = -2$~~
racine étrangère

$4x = \frac{-13-8}{4}$

$$3. \quad 3 \log a + \frac{1}{2} \log b - \log c - 2 \log d$$

a)

$$b) \quad \frac{2}{3} \ln a - 2 \ln b - \frac{3}{4} \ln c$$

$$4. \quad \log \frac{3^4 \cdot 2^2}{6^2}$$

$$a) = \log \frac{81 \cdot 4}{36}$$

$$\log 9$$

$$b) \quad \log_7 \frac{x^2}{y \sqrt[4]{z}}$$

$$5. \quad \log_3 27 = 3$$

$$\log_3 30 = x$$

$$\log_3 81 = 4$$

$$3 < x < 4$$

$$x \approx 3,1$$

$$\log_2 8 = 3$$

$$\log_2 9 = x$$

$$\log_2 16 = 4$$

$$3 < x < 4$$

$$x \approx 3,2$$

$$\log_4 16 = 2$$

$$\log_4 60 = x$$

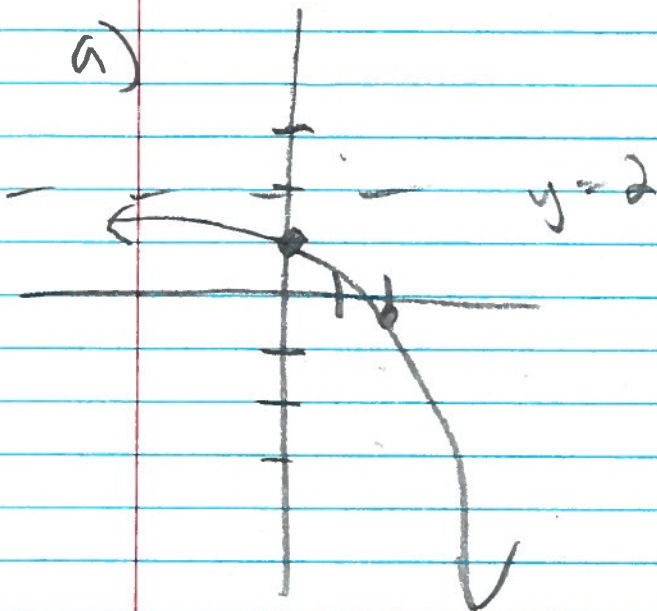
$$\log_4 64 = 3$$

$$2 < x < 3$$

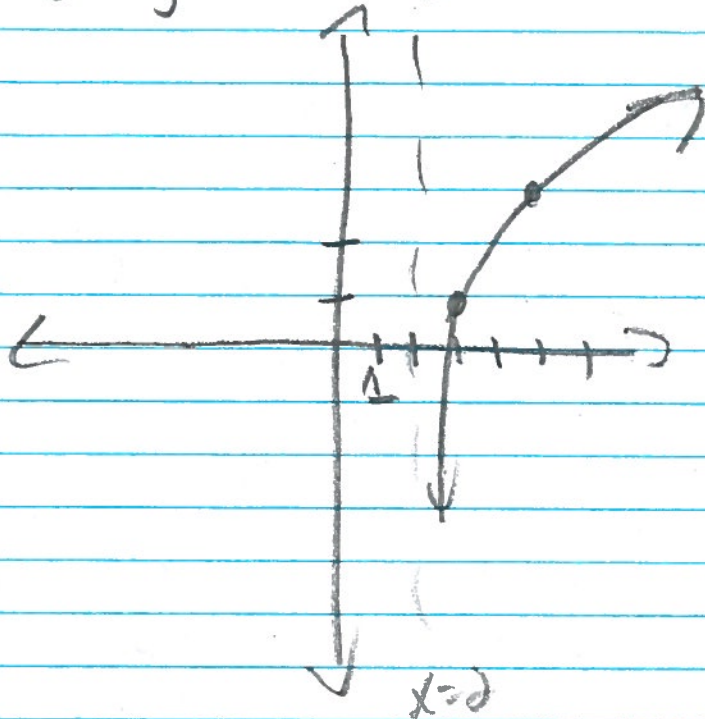
b. $y = e^x + 2$

$y = e^x$

a)



b) $y = \ln(x-2) + 1$



7.

a) $3 = \log_2(x+5)(x+7)$

$$2^3 = x^2 + 12x + 35$$

$$0 = x^2 + 12x + 27$$

$$0 = (x+9)(x+3)$$

~~$$x = -9 \quad x = -3$$~~

racine
étrangère

b) $\log_x(x+1) = \log(7x-8)$

$$x^2 + x = 7x - 8$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 4 \quad x = 2 \quad \checkmark$$

$$c) \frac{5(3^x)}{5} = \frac{60}{5}$$

$$3^x = 12$$

$$\log 3^x = \log 12$$

$$x \log 3 = \log 12$$

$$x \hat{=} \frac{\log 12}{\log 3}$$

$$x \hat{=} 2,262$$

$$d) \log 3^{x+4} = \log 5^{x+1}$$

$$(x+4) \log 3 = (x+1) \log 5$$

$$x \log 3 + 4 \log 3 = x \log 5 + \log 5$$

$$x \log 3 - x \log 5 = \log 5 - 4 \log 3$$

$$x(\log 3 - \log 5) = \log 5 - 4 \log 3$$

$$x = \frac{(\log 5 - 4 \log 3)}{\log 3 - \log 5}$$

$$x \hat{=} 5,452$$

$$8. \frac{\log 100}{\log 8}$$

$$a) \log 8$$

$$2,215$$

$$b) 2^x = 64$$

$$2^6 = 64$$

$$\log_2 64 = 6$$

$$c) \log_2 20$$

$$= \frac{\log 20}{\log 2}$$

$$\log 2$$

$$4,322$$

9. a) $\frac{4}{10} = \frac{10}{10} e^{k \cdot 5}$

b) $M = 10 e^{-0,045 \cdot 10}$
 $M = 6,376$

$$\frac{4}{5} = e^{k \cdot 5}$$

$$\ln\left(\frac{4}{5}\right) = \ln e^{k \cdot 5}$$

$$\ln\left(\frac{4}{5}\right) = 5k \ln e$$

$$\frac{\ln\left(\frac{4}{5}\right)}{5} = k$$

$$k = -0,045$$

c) $5 = 10 e^{-0,045 t}$

$$\frac{1}{2} = e^{-0,045 t}$$

$$\ln 0,5 = \ln e^{-0,045 t}$$

$$\frac{\ln 0,5}{-0,045} = t$$

$$t = 15,4103 \text{ ans}$$

10. décroissante parce que la valeur de la base est entre 0 et 1

