Calcul 42S

Unité : Maximum et Minimum Points Critiques : Mini Quiz Pts critiques et Pts d'inflexions des Fonctions Trigonométriques

Nom : _____ Date : _____

1. Trouve le max et min absolues des fonctions suivantes sur l'intervalle $[0, 2\pi]$.

a) $y = 3\sin 4x$

$n = 4x$

$[0, \frac{16\pi}{8}]$

$y' = 12\cos 4x$

$0 = 12\cos 4x$

$0 = \cos n$

$4x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$ $\quad 4x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{\pi}{8} + \frac{\pi}{2} n, n \in \mathbb{Z}$ $\quad x = \frac{3\pi}{8} + \frac{\pi}{2} n, n \in \mathbb{Z}$

$y\ max = 3$ $\quad y\ min = -3$

	0	$\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$	$\frac{16\pi}{8}$
y'	+	0 −	0	+ 0	−	0 +	0	− 0	+	0 − 0 +
y	↗	max ↘	min	↗ max	↘	min ↗	max	↘ min	↗	max ↘ min ↗

b) $y = \cos^2 2x$

$\min^0$ $(\frac{3\pi}{8}, -3), (\frac{7\pi}{8}, -3), (\frac{11\pi}{8}, -3), (\frac{15\pi}{8}, -3)$

$\max^0$ $(\frac{\pi}{8}, 3), (\frac{5\pi}{8}, 3), (\frac{9\pi}{8}, 3), (\frac{13\pi}{8}, 3)$

$y' = 2\cos 2x \cdot -\sin 2x \cdot 2$

$[0, \frac{8\pi}{4}]$

$y' = -4\cos 2x \sin 2x$

$0 = -4\cos 2x \sin 2x$

$\max^0$ $(0,1), (\frac{\pi}{2}, 1), (\pi, 1), (\frac{3\pi}{2}, 1), (2\pi, 1)$

$\cos 2x = 0$

$2x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$ $\quad x = \frac{\pi}{4} + \pi n, n \in \mathbb{Z}$

$2x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{3\pi}{4} + \pi n, n \in \mathbb{Z}$

$\sin 2x = 0$

$2x = 0 + 2\pi n$

$x = 0 + \pi n$

$2x = \pi + 2\pi n$

$\pi/2 + \pi n$

$2x = 2\pi + 2\pi n$

$\pi + \pi n$

$\frac{1}{2}\pi n \quad x = \frac{\pi}{2} n$

	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y'	0	− 0	+ 0	− 0	+ 0	− 0	+ 0	− 0	+ 0
y	max	↘ min	↗ max	↘ min	↗ max	↘ min	↗ max	↘ min	↗ max

$\min^0$ $(\frac{\pi}{4}, 0), (\frac{3\pi}{4}, 0), (\frac{5\pi}{4}, 0), (\frac{7\pi}{4}, 0)$

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2. Trouve les points d'inflexions des fonctions suivantes sur l'intervalle $[0, 2\pi]$.

c) $y = 3\sin 4x$ $y' = 12\cos 4x$ $[0, \frac{8\pi}{4}]$

$y'' = -12\sin 4x \cdot 4$

$0 = -48\sin 4x$

$0, \pi, 2\pi$ $4x = \pi n$

$4x = 0 + 2\pi n$ $4x = \pi + 2\pi n$ $4x = 2\pi + 2\pi n$

$x = \frac{\pi}{2}n$ $x = \frac{\pi}{4} + \frac{\pi}{2}n$ $x = \frac{\pi}{2} + \frac{\pi}{2}n$

$x = \frac{\pi}{4}n, n \in \mathbb{Z}$

	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y''	0 −	0 +	0 −	0 +	0 −	0 +	0 −	0 +	0
y	∩	∪	∩	∪	∩	∪	∩	∪	

$y = 0$

pts d'inflexions

$(\frac{\pi}{4}, 0), (\frac{\pi}{2}, 0), (\frac{3\pi}{4}, 0), (\pi, 0), (\frac{5\pi}{4}, 0), (\frac{3\pi}{2}, 0), (\frac{7\pi}{4}, 0), (2\pi, 0)$

d) $y = \cos^2 2x$ $[0, \frac{16\pi}{8}]$

$y' = -4\cos 2x \sin 2x$

$y'' = 8\sin 2x \sin 2x + 2\cos 2x \cdot -4\cos 2x$

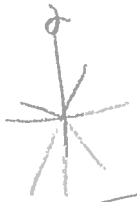
$y'' = 8\sin^2 2x - 8\cos^2 2x$ $y'' = 8\sin^2 x - 8(1 - \sin^2 x)$

$0 = 8\sin^2 2x - 8 + 8\sin^2 x$

$8 = \frac{16}{16}\sin^2 2x$

$\pm\sqrt{\frac{1}{2}} = \sqrt{\sin^2 2x}$

$\pm\frac{\sqrt{2}}{2} = \sin 2x$



$2x = \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{\pi}{8} + \pi n, n \in \mathbb{Z}$

$2x = \frac{5\pi}{4} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{5\pi}{8} + \pi n, n \in \mathbb{Z}$

$2x = \frac{3\pi}{4} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{3\pi}{8} + \pi n, n \in \mathbb{Z}$

$2x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{7\pi}{8} + \pi n, n \in \mathbb{Z}$

$x =$	$\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$
y''	− 0 +	0 −	0 +	0 −	0 +	0 −	0 +	0 − 0 +
y	∩	∪	∩	∪	∩	∪	∩	∪

$y = 0,5$