Mini Quiz integrale indefinie

1. a) $\int \frac{dx}{\sqrt{2x+7}}$

$u = 2x+7$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$\int \frac{du}{2\sqrt{u}}$

$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$\frac{1}{2}\,u^{-\frac{1}{2}+1} \div -\frac{1}{2}+1$

$\frac{1}{\cancel{2}}\,u^{\frac{1}{2}}\cdot\cancel{2}$

$\sqrt{2x+7} + K$

b) $\int \cos(3x+7)\,dx$

$u = 3x+7$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$\int \cos u\,\frac{du}{3}$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin(3x+7) + K$

c) $\int (x^2+1)^2 x\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int (u)^2\,\cancel{x}\,\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^2\,du$

$\frac{1}{2}\,\frac{u^3}{3}$

$\frac{(x^2+1)^3}{6} + K$

d) $\int x^2 e^{x^3}\,dx$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$\int \cancel{x^2}\,e^u\,\frac{du}{3\cancel{x^2}}$

$\frac{1}{3}\int e^u\,du$

$\frac{e^{x^3}}{3} + K$

e) $\int \frac{\sin x}{\cos^8 x}\, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \qquad \frac{du}{-\sin x} = dx$

$\int \frac{\cancel{\sin x}}{u^8} \cdot \frac{du}{-\cancel{\sin x}}$

$-\int u^{-8}\, du$

$-\frac{(u)^{-7}}{-7}$

$\frac{1}{7\cos^7 x} + K$ or $\frac{\sec^7 x}{7} + K$

f) $\int \frac{3x+2}{x+5}\, dx$

$$\begin{array}{r} 3 \\ x+5\,\overline{)\,3x+2} \\ -3x+15 \\ \hline -13 \end{array}$$

$\int \left(3 - \frac{13}{x+5}\right) dx$

$\int 3\, dx - \int \frac{13}{x+5}\, dx$

$3x - 13\ln|x+5| + K$

g) $\int (x^2+e^x)^2 (2x+e^x)\, dx$

$u = x^2 + e^x$

$\frac{du}{dx} = 2x + e^x$

$\frac{du}{2x+e^x} = dx$

$\int u^2 \cdot \cancel{(2x+e^x)} \cdot \frac{du}{\cancel{2x+e^x}}$

$\int u^2\, du = \frac{(x^2+e^x)^3}{3} + K$

h) $\int \cos x\, e^{\sin x}\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

$\int \cancel{\cos x}\, e^u \cdot \frac{du}{\cancel{\cos x}}$

$\int e^u\, du$

$e^{\sin x} + K$

i) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

$\displaystyle\int \frac{\cos u}{\cancel{x}} \cdot \cancel{x}\,du$

$\displaystyle\int \cos u\,du$

$\sin(\ln x) + K$

j) $\displaystyle\int \frac{6x+2}{\sqrt{3x^2+2x-5}}\,dx$

$u = 3x^2+2x-5$

$\frac{du}{dx} = 6x+2$

$\frac{du}{6x+2} = dx$

$\displaystyle\int \frac{\cancel{6x+2}}{u} \cdot \frac{du}{\cancel{6x+2}}$

$\displaystyle\int \frac{du}{u}$

$\ln(3x^2+2x-5) + K$

k) $\displaystyle\int \frac{6x+2}{(3x^2+2x-5)^2}\,dx$

$u = 3x^2+2x-5$

$\frac{du}{dx} = 6x+2$

$\frac{du}{6x+2} = dx$

$\displaystyle\int \frac{\cancel{6x+2}}{u^2} \cdot \frac{du}{\cancel{6x+2}}$

$\displaystyle\int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{3x^2+2x-5} + K$

l) $\displaystyle\int 2^{3x}\,dx$ ☆ $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + K$

$u = 3x$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$\displaystyle\int 2^u \frac{du}{3} = \frac{2^{3x}}{3\ln 2} + K$

m) $\displaystyle\int \frac{x^3+1}{x^2}\,dx$

$$\begin{array}{r|l} & x \\ \hline x^2 & x^3+1 \\ & -x^3 \\ \hline & 1 \end{array} \Rightarrow x + \frac{1}{x^2}$$

$$\int \left(x + \frac{1}{x^2}\right) dx$$

$$\int x\,dx + \int x^{-2}\,dx$$

$$\frac{x^2}{2} - \frac{1}{x} + K$$

n) $\displaystyle\int 4x^2\sqrt{x^3-1}\,dx$

$$u = x^3 - 1$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{du}{3x^2} = dx$$

$$\int 4\cancel{x^2}\,u^{\frac{1}{2}}\,\frac{du}{3\cancel{x^2}}$$

$$\frac{4}{3}\int u^{\frac{1}{2}}\,du$$

$$\frac{4}{3}\,(u)^{3/2} \div \frac{3}{2}$$

$$\frac{8}{9}\,(x^3-1)^{3/2} + K$$

2. a) $\frac{dy}{dx} = 2y\cos 2x$

$\int \frac{dy}{y} = \int \cos 2x \, dx$

$\ln y = \sin 2x + K$

b) $\frac{dy}{dx} = \frac{e^x}{e^y - e^{-y}}$

$\int dy\,(e^y - e^{-y}) = \int e^x \, dx$

$e^y - (-e^{-y}) = e^x + K$

$e^y + e^{-y} - e^x = K$

c) $e^x dx + \cos y \, dy = 0$

$\int e^x dx = \int -\cos y \, dy$

$e^x = -\sin y + K$

$e^x + \sin y = K$

d) $\frac{ye^x dx}{y} + \frac{(y+1)dy}{y} = \frac{0}{y}$

$\int e^x dx + \int \frac{(y+1)}{y} dy = 0$

$$\begin{array}{r} 1 \\ y \overline{\smash{)}\, y+1} \\ \underline{-y } \\ 0+1 \end{array} = 1 + \frac{1}{y}$$

$\int e^x dx + \int \left(1 + \frac{1}{y}\right) dy$

$e^x + y + \ln y = K$

2.

e) $x\sqrt{1-y}\,dx - \sqrt{1-x^2}\,dy = 0$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx - \int \frac{dy}{\sqrt{1-y}} = 0$$

$$\boxed{2\sqrt{1-y} = \sqrt{1-x^2} + k}$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

$$\int \frac{\cancel{x}}{u^{\frac{1}{2}}} \cdot \frac{du}{-2\cancel{x}}$$

$$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$-\frac{1}{\cancel{2}}\, u^{\frac{1}{2}} \cdot \cancel{2}$

$-\sqrt{(1-x^2)}$

$$\int \frac{dy}{\sqrt{1-y}}$$

$u = 1-y$

$\frac{du}{dy} = -1$

$-du = dy$

$$\int \frac{-du}{u^{\frac{1}{2}}}$$

$$-\int u^{-\frac{1}{2}}\,du$$

$-u^{\frac{1}{2}} \cdot 2$

$-2\sqrt{1-y}$

$-\sqrt{1-x^2} - (-2\sqrt{1-y}) = k$

$$\boxed{2\sqrt{1-y} = \sqrt{1-x^2} + k}$$

2.

F) $dx + (y^2 + xy^2)dy = 0$

$$dx + y^2(1+x)dy = 0$$

$$\frac{dx}{1+x} + y^2 dy = 0$$

$$\ln(1+x) + \frac{y^3}{3} + K = 0$$

$$3\ln(1+x) + y^3 + K = 0$$

3. pente $= x^2 + x + 1 \rightarrow$ dérivé

$$\frac{dy}{dx} = x^2 + x + 1$$

$$dy = (x^2 + x + 1)\,dx$$

$$\int dy = \int (x^2 + x + 1)\,dx$$

$$y = \frac{x^3}{3} + \frac{x^2}{2} + x + K$$

a) passe par l'origine (0,0)

$$0 = \frac{0^3}{3} + \frac{0^2}{2} + 0 + K$$

$K = 0$ l'équation est donc.

$$y = \frac{x^3}{3} + \frac{x^2}{2} + x$$

b) passe par le point (1,2)

$$2 = \frac{1^3}{3} + \frac{1^2}{2} + 1 + K$$

$$1 \cdot 6 = \frac{1}{3} \cdot 6 + \frac{1}{2} \cdot 6 + K \cdot 6$$

$6 = 2 + 3 + 6K \quad K = 1/6$

$$y = \frac{x^3}{3} + \frac{x^2}{2} + x + \frac{1}{6}$$

$$\boxed{6y = 2x^3 + 3x^2 + 6x + 1}$$

4. 1 km

5. 99,04 m/s

6. 155 grammes

4. $a = -0{,}8 m/s^2$

$a = \frac{\Delta v}{\Delta t}$ $\quad v_i = 40 m/s$ décélère et $v_F = 0 m/s$

$\Delta t = \frac{\Delta v}{a}$

$\Delta t = \frac{0 - 40}{-0{,}8}$

$\Delta t = 50$ sec.

$v = \int -0{,}8\, dt$

$v = -0{,}8t + k_1$

$v = -0{,}8(50) + k_1 \quad 0 = -40 + k$

$40 = k_1$

$v = -0{,}8t + 40$

$d = \int (-0{,}8t + 40)\, dt$

$d = -\frac{0{,}8t^2}{2} + 40t + k_2$

$d = -0{,}4t^2 + 40t + k_2$

initialement $d = 0$ $t = 0$

$0 = -0{,}4(0)^2 + 40(0) + k_2$

$k_2 = 0$

$d = -0{,}4t^2 + 40t$

$d = -0{,}4(50)^2 + 40(50)$

$d = 1000 m$

$\boxed{d = 1 km}$

5.

$a = -9{,}8\ m/s^2 = \frac{dv}{dt} = -9{,}8$

$v = -9{,}8t + k_1$

initialement $t = 0$ et vitesse initiale $= 3\ m/s$

$3 = -9{,}8(0) + k_1$

$3 = k_1$

$v = -9{,}8t + 3 = \frac{dy}{dt}$

y = hauteur/distance verticale

$y = -\frac{9{,}8t^2}{2} + 3t + k_2$

$500 = -4{,}9(0)^2 + 3(0) + k_2$

$500 = k_2$

$y = -4{,}9t^2 + 3t + 500$

quand on échappe la balle à 500 mètres (distance initiale) le temps initiale égale 0.
On doit trouver le temps finale quand la balle touche le sol.

$t = \frac{-3 \pm \sqrt{(3)^2 - 4(-4{,}9)\cdot 500}}{2(-4{,}9)}$

$t = \frac{-3 \pm \sqrt{9809}}{-9{,}8}$

$t = \cancel{-9{,}800}$ $\boxed{t = 10{,}412}$

vitesse avant de toucher le sol ($t = 10{,}412$ sec)

$v = -9{,}8(10{,}412) + 3$

$v = -99{,}04\ m/s$

$v = 99{,}04\ m/s$ vers le bas

6. $Q \rightarrow$ quantité de sel

$\frac{dQ}{dt}$ taux de variation = (ce qui entre) − (ce qui sort)

$$\frac{dQ}{dt} = 0{,}6 \times 5 - \frac{Q}{400} \cdot 5$$

$\frac{dQ}{dt}$ = quantité de sel en fct du temps

$$\frac{dQ}{dt} = 3 - \frac{Q}{80}$$

$$\frac{dQ}{dt} = \frac{240 - Q}{80}$$

$$\frac{80\,dQ}{240 - Q} = dt$$

$$\int \frac{80\,dQ}{240 - Q} = \int dt$$

$$80 \int \frac{dQ}{240 - Q} = t + K$$

$$u = 240 - Q$$

$$\frac{du}{dQ} = -1$$

$$-du = dQ$$

$$80 \int \frac{-du}{240 - Q} = t + K$$

$$-80 \ln|240 - Q| = t + K$$

$$\ln|240 - Q| = \frac{t + K}{-80}$$

$$e^{\frac{-t-K}{80}} = 240 - Q$$

$$Q = 240 - e^{\frac{-t-K}{80}}$$

$$Q = 240 - e^{\frac{-t}{80}} e^{\frac{-K}{80}}$$

Q initiale est 100 grammes au temps $t = 0$ sec.

$$100 = 240 - e^{\frac{-0}{80}} e^{\frac{-K}{80}}$$

$$-140 = -1 e^{\frac{-K}{80}} \qquad 140 = e^{\frac{-K}{80}}$$

substitue $e^{\frac{-K}{80}}$ dans la formule de Q au $t = 40$ min

$$Q = 240 - 140 e^{\frac{-40}{80}}$$

$Q = 155$ grammes