Calcul 42S
Unité : Les Intégrales : Mini Quiz Les Intégrales définies

Nom : ______________________ Date : ________________________

1. Détermine les intégrales définies.

a)

$$\int_0^2 (3x^3 - 2x + 5)\,dx$$

(18)

$$\left.\frac{3x^4}{4} - x^2 + 5x\right|_0^2$$

$$\left(\frac{3(2)^4}{4} - 2^2 + 5(2)\right) - \left(\frac{3(0)^4}{4} - 0^2 + 5(0)\right)$$

$$(12 - 4 + 10) - 0)$$

$$= 18$$

b)

$$\int_0^1 x\,(2x^2 - 1)^{10} dx$$

(1/22)

$u = 2x^2 - 1$

$\frac{du}{4x} = dx$

$$\int_0^1 \cancel{x}(u)^{10}\frac{du}{4\cancel{x}} = \frac{1}{4}\int_0^1 u^{10}\,du$$

$$\left.\frac{(2x^2-1)^{11}}{44}\right|_0^1 = \left[\frac{(2(1)^2-1)^{11}}{44}\right] - \left[\frac{(2(0)^2-1)^{11}}{44}\right]$$

$$= \frac{1}{44} - \left(\frac{-1}{44}\right) = \frac{2}{44}$$

$$= \frac{1}{22}$$

c)

$$\int_{-1}^1 \frac{2x}{x^2 - 1}\,dx$$

$u = x^2 - 1$

$\frac{du}{2x} = dx$

$$\int_{-1}^1 \frac{\cancel{2x}\,du}{u\,\cancel{2x}} = \int_{-1}^1 \frac{du}{u}$$

$$\ln|x^2 - 1|\Big|_{-1}^1 = \ln|1^2 - 1| - \ln|(-1)^2 - 1|$$

$$= \ln 0 - \ln 0$$

= indéfini

d)

$$\int_{\pi/6}^{\pi/2} \sin^2 x \cos x\,dx$$

(7/24)

$u = \sin x$

$\frac{du}{\cos x} = dx$

$$\int_{\pi/6}^{\pi/2} u^2 \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = \int_{\pi/6}^{\pi/2} u^2\,du$$

$$= \left.\frac{\sin^3 x}{3}\right|_{\pi/6}^{\pi/2} = \left(\frac{\sin^3\frac{\pi}{2}}{3}\right) - \left(\frac{\sin^3\frac{\pi}{6}}{3}\right)$$

$$= \frac{1}{3} - \frac{1}{24}$$

$$= \frac{8-1}{24} = \frac{7}{24}$$

e)

$$\int_0^1 x^3(1-x)^2\,dx = \int_0^1 x^3(1-2x+x^2)$$

(1/60)

$$\int_0^1 (x^3-2x^4+x^5)\,dx$$

$$= \frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\Big|_0^1$$

$$= \left(\frac{1^4}{4} - \frac{2(1)^5}{5} + \frac{1^6}{6}\right) - (0-0+0)$$

$$= \frac{15-24+10}{60} - 0 = 1/60$$

f)

$$\int_1^e \left(-\frac{3}{x}+2\right)dx$$

(2e – 5)

$$= -3\ln|x| + 2x\Big|_1^e$$

$$= (-3\ln e + 2e) - (-3\ln 1 + 2(1))$$

$$= (-3+2e) - (0+2)$$

$$= 2e-5$$

g)

$$\int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx = \int_0^1 \frac{x\ dx}{\sqrt{u}}$$

$(2 - \sqrt{3})$

$$u = 4-x^2$$

$$\frac{du}{-2x} = dx$$

$$\int_0^1 \frac{\cancel{x}}{\sqrt{u}}\frac{du}{-2\cancel{x}} = -\frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{-1u^{\frac{1}{2}}}{\cancel{2}} \cdot \cancel{2}\Big|_0^1$$

$$= -\sqrt{4-x^2}\Big|_0^1$$

$$= \left(-\sqrt{4-(1)^2}\right) - \left(-\sqrt{4-(0)^2}\right)$$

$$= -\sqrt{3} + 2 = 2-\sqrt{3}$$

h)

$$\int_1^6 \frac{x\,dx}{\sqrt{(3+x)}} \qquad u = \sqrt{3+x}$$

$$u^2 = 3+x$$

$$u^2-3 = x$$

$$2u\,du = dx$$

$$\int_1^6 \frac{(u^2-3)\,2\cancel{u}\,du}{\cancel{u}} = 2\int_1^6 (u^2-3)\,du$$

$$\frac{2u^3}{3} - 6u\Big|^6 = \frac{2(\sqrt{3+x})^{3/2}}{3} - 6\sqrt{3+x}$$

$$\frac{2}{3}(3+x)^{\frac{1}{2}}(3+x-9)$$

$$= \frac{2}{3}\sqrt{3+x}\,(x-6)\Big|_1^6$$

$$\frac{2}{3}\sqrt{3+6}\,(6-6) - \left(\frac{2}{3}\sqrt{3+1}\,(1-6)\right)$$

$$0 - \left(\frac{4}{3}\cdot -5\right) = \frac{20}{3}$$

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i) $\int_{\pi}^{5\pi/4} \tan x\,dx + \int_{-\pi}^{\pi} \cos x\,dx$

$\ln|\sec u|\Big|_{\pi}^{5\pi/4} + \sin x\Big|_{\pi}^{\pi}$

$\left(\ln|\sec\tfrac{5\pi}{4}| - \ln|\sec\pi|\right) + \left(\sin\pi - \sin(-\pi)\right)$

$\left(\ln|-\tfrac{2}{\sqrt2}| - \ln|-1|\right) + (0-0)$

$\ln\frac{2}{\sqrt2} - \ln 1 = \ln\frac{2}{\sqrt2}$

$= \ln 2 - \ln\sqrt2$

j) $\int_1^8 \frac{\sqrt{x}-x^2}{\sqrt[3]{x}}\,dx$ $\quad \int_1^8 \left(\frac{x^{\frac12}}{x^{\frac13}} - \frac{x^2}{x^{\frac13}}\right)dx$

(-5403/56)

$\int_1^8 \left(x^{\frac12-\frac13} - x^{2-\frac13}\right)dx = \int_1^8 \left(x^{\frac16} - x^{\frac56}\right)dx$

$= \left(\frac{6x^{\frac76}}{7} - \frac{6x^{\frac{11}{6}}}{11}\right)\Big|_1^8$

$= \left(\frac{6\cdot 8^{\frac76}}{7} - \frac{6\cdot 8^{\frac{11}{6}}}{11}\right) - \left(\frac{6(1)^{\frac76}}{7} - \frac{6(1)^{\frac{11}{6}}}{11}\right)$

k) $\int_3^6 (1 - y + y^2)\,dy$ (52,5)

$\left(y - \frac{y^2}{2} + \frac{y^3}{3}\right)\Big|_3^6$

$= \left(6 - \frac{6^2}{2} + \frac{6^3}{3}\right) - \left(3 - \frac{3^2}{2} + \frac{3^3}{3}\right)$

$= 60 - 7,5$

$= 52,5$

l) $\int_1^3 \left(\frac{1}{x^2} + \frac{1}{x^3}\right)dx$ (10/9)

$\int_1^3 \left(x^{-2} + x^{-3}\right)dx$

$= \left(\frac{x^{-1}}{-1} + \frac{x^{-2}}{-2}\right)\Big|_1^3$

$= -\frac{1}{x} - \frac{1}{2x^2}\Big|_1^3$

$= \left(-\frac{1}{3} - \frac{1}{2(3)^2}\right) - \left(-\frac{1}{1} - \frac{1}{2(1)^2}\right)$

$= -\frac{7}{18} - \left(-\frac{3}{2}\right) = \frac{20}{18} = \boxed{\frac{10}{9}}$

m)

$$\int_{-1}^{1} e^{-4x}\,dx \qquad \int_{-1}^{1} e^{u}\,dx$$

$u=-4x$

$\frac{du}{-4}=dx$

$$\int_{-1}^{1} e^{u}\cdot\frac{du}{-4} = -\frac{1}{4}\int_{-1}^{1} e^{u}\,du$$

$$-\frac{e^{-4x}}{4}\Bigg|_{-1}^{1} = \left(-\frac{e^{-4\cdot 1}}{4}\right)-\left(-\frac{e^{-4\cdot -1}}{4}\right)$$

$= 13{,}645$

n)

$$\int_{1}^{2} \frac{\sqrt{1+\ln x}}{x}\,dx \qquad \int_{1}^{2} \frac{\sqrt{u}}{\cancel{x}}\cdot \cancel{x}du$$

$u=1+\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$x\,du=dx$

$$\int_{1}^{2}\sqrt{u}\,du$$

$$\frac{2u^{3/2}}{3}\Bigg|_{1}^{2} = \frac{2(1+\ln x)^{3/2}}{3}$$

$$=\left(\frac{2(1+\ln 2)^{3/2}}{3}\right)-\left(\frac{2(1+\ln 1)^{3/2}}{3}\right)$$

$=0{,}802$

2. Suppose que $\int_0^2 f(t)dt = 5$, $\int_2^5 f(t)dt = 6$ et $\int_0^7 f(t)dt = 3$

a) Trouve $\int_0^5 f(t)dt$

$$\int_2^5 F(t)dt - \int_0^2 F(t)dt = \int_0^5 f(t)dt$$

$= 6-5 = 1$

b) Trouve $\int_5^7 f(t)dt$

$$\int_0^7 F(t)dt - \int_0^5 F(t)dt = \int_5^7 F(t)dt$$

$= 3-1 = 2$