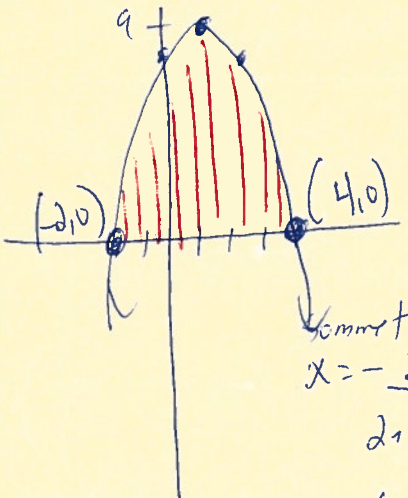
Calcul 42S

Unité : Les Intégrales : Mini Quiz Les aires entres les courbes

Nom : ______________________________ Date : ____________________________________

1. Trouve l'aire bornée par les courbes.

a) $y = -x^2 + 2x + 8$ et $y = 0$



abs $0 = -x^2 + 2x + 8$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4 \quad x = -2$

ord. $y = 8$

Sommet

$x = -\frac{2}{2 \cdot -1} = 1$

$y = -(1)^2 + 2(1) + 8 = 9$

$$\int_{-2}^{4} \left[(-x^2 + 2x + 8) - 0\right] dx$$

$$\left. -\frac{x^3}{3} + x^2 + 8x - 0 \right|_{-2}^{4}$$

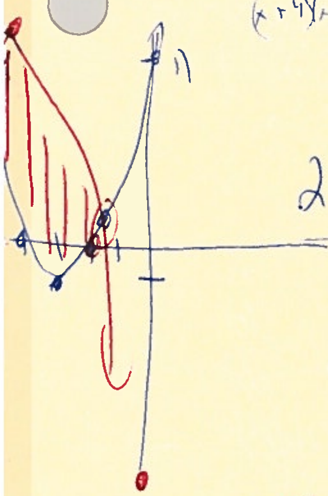
$$\left(-\frac{(4)^3}{3} + 4^2 + 8(4)\right) - \left(-\frac{(-2)^3}{3} + (-2)^2 + 8(-2)\right)$$

$$-\frac{64}{3} + 16 + 32 - \left(+\frac{8}{3} + 4 - 16\right)$$

$$\frac{80}{3} - \left(-\frac{28}{3}\right) = \frac{108}{3} = 36$$

b) $y = x^2 + 6x + 8$ et $y = -x^2 - 10x - 16$

1) $(x+4)(x+2)$



$x^2 + 6x + 8 = -x^2 - 10x - 16$

$$\frac{2x^2 + 16x + 24 = 0}{2}$$

$x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x = -6 \quad x = -2$

$$\int_{-6}^{-2} \left[(-x^2 - 10x - 16) - (x^2 + 6x + 8)\right] dx$$

$$\left. \left(-\frac{x^3}{3} - 5x^2 - 16x\right) - \left(\frac{x^3}{3} + 3x^2 + 8x\right) \right|_{-6}^{-2}$$

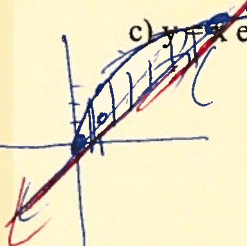
$$-\frac{(-2)^3}{3} - 5(-2)^2 - 16(-2) - \left(\frac{(-2)^3}{3} + 3(-2)^2 + 8(-2)\right) = 21{,}333$$

$$\left(-\frac{(-6)^3}{3} - 5(-6)^2 - 16(-6)\right) - \left(\frac{(-6)^3}{3} + 3(-6)^2 + 8(-6)\right) = 0$$

$$-12 - (-12) = 0$$

$$(21{,}333) - (0) = \boxed{21{,}333 \text{ ou } \frac{64}{3}}$$

c) $y = x$ et $y = 3\sqrt{x}$



$(x = 3\sqrt{x})^2$

$x^2 = 9 \cdot x$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0 \quad x = 9$

$$\int_{0}^{9} (3\sqrt{x} - x)\, dx$$

$$\left. 2x^{3/2} - \frac{x^2}{2} \right|_{0}^{9}$$

$$\left(2(9)^{3/2} - \frac{9^2}{2}\right) - \left(2(0)^{3/2} - \frac{0^2}{2}\right) = \boxed{\frac{27}{2} \text{ ou } 13{,}5}$$

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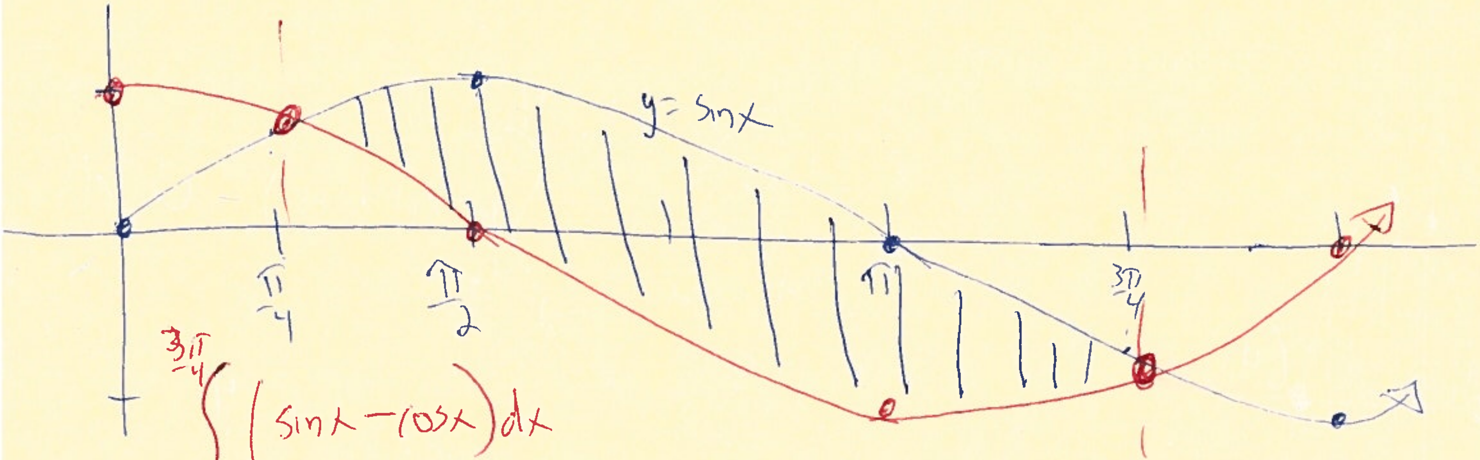
d) y = sinx et y = cosx entre $x = \frac{\pi}{4}$ et $x = \frac{3}{4}\pi$



$$\int_{\pi/4}^{3\pi/4} (\sin x - \cos x)\,dx$$

$$= \left(-\cos x - (\sin x)\right)\Big|_{\pi/4}^{3\pi/4}$$

$$= \left(-\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$= \left(-\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - \left(-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right)$$

$$\boxed{= \sqrt{2}}$$

$\sin x = \cos x$

$x = \frac{\pi}{4}$ et $\frac{3\pi}{4}$

e) $x^3 - 4x^2 - x + 4$, $y = 0$, $x = -1$, $x = 2$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -1 & 4 \\ & \downarrow & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$(x-1)(x^2-3x-4)$

$(x-1)(x-4)(x+1)=0$

$x=1$ $x=-1$ $x=4$

$$\int_{-1}^{1}\left((x^3-4x^2-x+4)-0\right)dx + \int_{1}^{2}\left(0-(x^3-4x^2-x+4)\right)dx$$

$-x^3+4x^2+x-4$

$$\frac{x^4}{4} - \frac{4x^3}{3} - \frac{x^2}{2} + 4x\Big|_{-1}^{1} + -\frac{x^4}{4} + \frac{4x^3}{3} + \frac{x^2}{2} - 4x\Big|_{1}^{2}$$

$$\left(\frac{1^4}{4} - \frac{4(1)^3}{3} - \frac{1^2}{2} + 4(1)\right) - \left(\frac{(-1)^4}{4} - \frac{4(-1)^3}{3} - \frac{(-1)^2}{2} + 4(-1)\right) + \left(\frac{-2^4}{4} + \frac{4(2)^3}{3} + \frac{2^2}{2} - 4(2)\right) - \left(\frac{-1^4}{4} + \frac{4(1)^3}{3} + \frac{1^2}{2} - 4(1)\right)$$

$$\left[\left(\frac{29}{12}\right) - \left(\frac{-35}{12}\right)\right] + \left[\left(\cancel{\frac{26}{3}}\ \frac{104}{12}\right) - \left(\frac{-23}{12}\right)\right]$$

$$\frac{64}{12} + \frac{127}{12} = \frac{191}{12}$$

ou 15,917