

Calcul 42S : Exercices dérivation des fonctions composées et Implicite

Trouver  $dy/dx$  si :

1  $y = 3u + 6$  et  $u = 2x^2 - 6x + 5$

2  $y = \sqrt{u^3}$  et  $u = 6x^3 - x + 11$

3  $y = 4v + v^2$  et  $v = x^4 + x^3 - x^2 + 4x - 1$

4  $y = 5w^2 + 7$  et  $w = 4x^5 - 2x^2 + 8x + 3$

5  $y = s + \sqrt{s}$  et  $s = \frac{x+1}{x-1}$

6  $y = r^2 + r$  et  $r = x^3 + 1 - \frac{1}{x}$

7  $y = \frac{1}{u+1}$  et  $u = \frac{1}{x+1}$

8  $y = \sqrt[3]{v}$  et  $v = \frac{5+x}{x-2}$

9  $y = \frac{w}{1+w}$  et  $w = x^2 + 7x - \frac{1}{3}$

10  $y = s\sqrt{s}$  et  $s = x\sqrt[3]{x^2}$

11  $y = r + r^{3/4}$  et  $r = x^{1/2} + x^{5/6}$

12  $y = 5u^2 - 3u$  et  $u = \frac{x^2+1}{x-9}$

13  $y = \frac{u}{2u-3}$  et  $u = \frac{x^2+3x-1}{3x-16}$

14  $y = 3u^2$ ;  $u = 4t - 1$  et  $t = -3x^2 + 6x - 16$

15  $y = w^3$  et  $w = \sqrt{x}$

16  $y = 5u^2 + 7u + 5$  et  $u = x^3 + x^2 - 7x + 3$

17  $y = t^3 + 6t - 5$  et  $x = t^2 + t + 3$

18  $y = 5t + \sqrt{t}$  et  $x = 2 + \sqrt{t}$

19  $y = 5t^4 + 2t^3 + t$  et  $x = \frac{1}{t}$

20  $y = \frac{2t+3}{t-1}$  et  $x = \frac{t^2+1}{t+1}$

21  $y = 3t^4 - 2t + 6$  et  $x = 4t - \sqrt{t}$

22  $y = \frac{t^2}{t+4}$  et  $x = \frac{4t+3}{t+4}$

23  $y = 4s^3 + 6s$  et  $x = 5s + 7$

24  $y = r^{3/4}$  et  $r = \frac{2r-3}{r+4}$

36  $y = \left( \frac{x^3 - 3x + 2}{2x^2 + 5x} \right)^4$

37  $y = (4x + 6)^4(2x - 5)^3$

38  $y = (x^2 + x + 2)^2(5x - 7)^3$

39  $y = (3x^3 + 6x^2 - 5)^3(4x^2 + 5)^4$

40  $y = (x^2 - 8)^3(2x^3 - 6x^2 + 7)^4$

41  $y = (3x^2 + 7)^2(x^8 - 2x^2 + 5)^3$

42  $y = \sqrt{x^2 + x}(2x + 5)^3$

43  $y = (x + 1)^3(2x + 9)^2(3x - 4)^4$

44  $y = (x^3 + 4x)^2(5x - 3)^5(x^2 + 8)^3$

45  $y = \sqrt{16 - 3x}(4x - 7)^2$

46  $y = \left( \frac{x^2 - 6}{3x + 4} \right)^3$

47  $y = \frac{(x - 5)^3}{(2x + 7)^4}$

48  $y = \frac{(x^3 - 2x)^2}{x^2 + 4}$

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$$\textcircled{24} \quad y = r^{3/4} \quad \text{et} \quad x = \frac{2r-3}{r+1}$$

$$\textcircled{25} \quad y = \frac{4t+3}{t^2+3} \quad \text{et} \quad x = \frac{1}{t-7}$$

$$\textcircled{26} \quad y = (x^2 + 5x + 6)^4$$

$$\textcircled{27} \quad y = (5x^3 - 8x + 1)^7$$

$$\textcircled{28} \quad y = (7x - 15)^8$$

$$\textcircled{29} \quad y = (x + 2\sqrt{x})^5$$

$$\textcircled{30} \quad y = \left( \frac{x-3}{2x+7} \right)^3$$

$$\textcircled{31} \quad y = (x^5 + x^4 - 2x + 7)^{10}$$

$$\textcircled{32} \quad y = [(4x^3 - x^2 + 5x - 1)^3]^4$$

$$\textcircled{33} \quad y = \sqrt{(x^2 + 11x + 3)^9}$$

$$\textcircled{34} \quad y = \sqrt{\frac{2x+17}{x+3}}$$

$$\textcircled{35} \quad y = \left( \frac{x^2 + 5x - 1}{3x - 4} \right)^3$$

$$\textcircled{48} \quad y = \frac{(x^3 - 2x)^2}{(5x - 1)^4}$$

$$\textcircled{49} \quad y = \frac{\sqrt{9x-4}}{(3x+8)^3}$$

$$\textcircled{50} \quad y = \frac{(4x-5)^2}{\sqrt{5+x}}$$

$$\textcircled{51} \quad y = \left( \frac{x+3}{\sqrt{x}} \right)^3$$

$$\textcircled{52} \quad y = \left( 3x + 1 + \frac{1}{\sqrt{x}} \right)^5$$

$$\textcircled{53} \quad y = \left( x^2 + \frac{2x-3}{x+1} \right)^3$$

$$\textcircled{54} \quad y = \frac{\sqrt{x+6}}{\sqrt[3]{x-1}}$$

$$\textcircled{55} \quad y = \frac{\sqrt{3x+2} (x^2 - 7)}{6x + 5}$$

$$\textcircled{56} \quad y = \frac{\sqrt{x+3} (3x-11)}{\sqrt{2x+5} (4x-3)}$$



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Pour les numéros 57 à 64, trouver  $f^{-1}(y)$  et indiquer s'il s'agit d'une fonction. Trouver  $I_f$ .

57  $y = 2x + 9$

58  $y = 3x^2 - 5$

59  $y = \frac{x+2}{x-3}$

60  $y = 8x - 5$

61  $y = x^3 + 1$

62  $y = \sqrt{x^2 - 1}$

63  $y = \frac{5x+5}{x-5}$

64  $y = 2 + \sqrt{x}$

78  $x\sqrt{y} + y\sqrt{x} + xy = 3$

79  $x^3y^2 + 5xy - 4x^2 + 6y = 11$

80  $xy + 3 = \frac{x + 2y^2}{xy - 4x}$

81  $xy^3 + 3y + 6x^3 + 4 = 0$

82  $2xy^2 + x^2y - 6y - 5 = 0$

83  $x^3 + 5x + x^3y^3 - 1 = 0$

84  $xy + x^2y^2 + x^3y^3 = 8$

85  $\sqrt{x} + 7xy + \sqrt{y} = 9$

86  $x^8y^7 - 3 = 0$

87  $\frac{y}{x} + xy = 16$

Pour les numéros 65 à 87, trouver  $\frac{dy}{dx}$ .

65  $x = y^3 + 6y$

66  $x = y + 3\sqrt{y}$

67  $x = \frac{y+2}{2y-1}$

68  $x = \frac{y^2 + 2y + 6}{y^3 - y^2}$

69  $x = y^3 + 5y^2$

70  $x = \frac{1}{y+3}$

71  $x = \frac{1}{y^3+1}$

72  $x = \frac{y^3-3}{2y+3}$

73  $x = \sqrt{y^2 + 2y + 2}$

74  $x^2y^2 + y^2 - 6 = 0$

75  $xy^3 + 3y^2 - 6x = 0$

76  $x^2y^2 + x^3y^3 - 4x + 8y = 0$

77  $2x + 5xy + 8x^2 - 16y^2 + 11 = 0$

88 Trouver la pente de la tangente au point (1, 2) de la courbe  $x^2y^2 - y + x^3 = 3$ .

89 Trouver la pente de la tangente au point  $(2, 2\sqrt{3})$  du cercle  $x^2 + y^2 = 16$ .

90 Trouver le domaine de  $f$ , la fonction dérivée  $f'$  et le domaine de  $f'$  si :

a)  $f(x) = \sqrt{x-1}$

b)  $f(x) = \frac{5x+2}{3x-4}$

c)  $f(x) = |x+2|$

91 Trouver :

a)  $f'(2)$  si  $f(x) = (2x+6)^3(x^2-1)^2$

b)  $f'(0)$  si  $f(x) = \sqrt{x}$

92 Trouver  $\frac{dy}{dx}$  au point où  $x = 2$  si :

a)  $y = 4t^3 - 3t^2$  et  $t = x^3 + 1$

b)  $y = 4u + 3\sqrt{u}$  et  $u = 8x^2 - 7x - 10$

93 Trouver la pente de la tangente au point (2, 1) de la courbe représentative de la fonction

$$y = \left( \frac{x+3}{x^2+1} \right)^3$$

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Réponses:

1.  $12x - 18$

2.  $\frac{3}{2}(18x^2 - 1)\sqrt{6x^3 - x + 11}$

3.  $(2x^4 + 2x^3 - 2x^2 + 8x + 2)(4x^3 + 3x^2 - 2x + 4)$

4.

$$\frac{dy}{dw} = 10w \text{ et } \frac{dw}{dx} = 20x^4 - 4x + 8$$

$$\frac{dy}{dx} = 10w(20x^4 - 4x + 8) = 40w(5x^4 - x + 2)$$

$$= 40(4x^5 - 2x^2 + 8x + 3)(5x^4 - x + 2)$$

5.

$$\frac{dy}{ds} = 1 + \frac{1}{2\sqrt{s}} \text{ et } \frac{ds}{dx} = \frac{-2}{(x-1)^2}$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{2\sqrt{s}}\right) \left(\frac{-2}{(x-1)^2}\right) = \left(1 + \frac{1}{2\sqrt{\frac{x+1}{x-1}}}\right) \left(\frac{-2}{(x-1)^2}\right)$$

6.

$$\frac{dy}{dr} = 2r + 1 \text{ et } \frac{dr}{dx} = 3x^2 + \frac{1}{x^2}$$

$$\begin{aligned} \frac{dy}{dx} &= (2r + 1) \left(3x^2 + \frac{1}{x^2}\right) = \left(2x^3 + 2 - \frac{2}{x} + 1\right) \left(3x^2 + \frac{1}{x^2}\right) \\ &= \frac{(2x^4 + 3x - 2)(3x^4 + 1)}{x^3} \end{aligned}$$

7.

$$\frac{dy}{du} = \frac{-1}{(u+1)^2} \text{ et } \frac{du}{dx} = \frac{-1}{(x+1)^2}$$

$$\frac{dy}{dx} = \left(\frac{-1}{(u+1)^2}\right) \left(\frac{-1}{(x+1)^2}\right) = \frac{1}{(u+1)^2(x+1)^2} = \frac{1}{(x+2)^2}$$

8.

$$\frac{dy}{dv} = \frac{1}{3}v^{-2/3} = \frac{1}{3v^{2/3}} \text{ et } \frac{dv}{dx} = \frac{1(x-2) - 1(5+x)}{(x-2)^2} = \frac{-7}{(x-2)^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\frac{1}{3v^{2/3}}\right) \left(\frac{-7}{(x-2)^2}\right) = \frac{-7}{3\left(\frac{5+x}{x-2}\right)^{2/3} (x-2)^2} \\ &= \frac{-7}{3(5+x)^{2/3} (x-2)^{4/3}} \end{aligned}$$

9.

$$\frac{dy}{dw} = \frac{1}{(1+w)^2} \text{ et } \frac{dw}{dx} = 2x + 7$$

$$\frac{dy}{dx} = \left(\frac{1}{(1+w)^2}\right) (2x + 7) = \frac{2x + 7}{\left(x^2 + 7x + \frac{2}{3}\right)^2}$$

10.

$$\frac{dy}{ds} = \frac{3\sqrt{s}}{2} \text{ et } \frac{ds}{dx} = \frac{5}{3}x^{2/3}$$

$$\frac{dy}{dx} = \left(\frac{3\sqrt{s}}{2}\right) \left(\frac{5}{3}x^{2/3}\right) = \frac{5}{2}\sqrt{s} x^{2/3} = \frac{5}{2}x^{5/6} x^{2/3} = \frac{5}{2}x^{3/2}$$

11.

$$\frac{dy}{dr} = 1 + \frac{3}{4}r^{-1/4} \text{ et } \frac{dr}{dx} = \frac{1}{2}x^{-1/2} + \frac{5}{6}x^{-1/6}$$

$$\frac{dy}{dx} = \left(1 + \frac{3}{4}r^{-1/4}\right) \left(\frac{1}{2}x^{-1/2} + \frac{5}{6}x^{-1/6}\right)$$

$$\frac{dy}{dx} = \left[1 + \frac{3}{4}(x^{1/2} + x^{5/6})^{-1/4}\right] \left[\frac{1}{2}x^{-1/2} + \frac{5}{6}x^{-1/6}\right]$$

12.

$$\frac{dy}{du} = 10u - 3 \text{ et } \frac{du}{dx} = \frac{x^2 - 18x - 1}{(x-9)^2}$$

$$\begin{aligned} \frac{dy}{dx} &= (10u - 3) \left[\frac{x^2 - 18x - 1}{(x-9)^2}\right] \\ &= \left[\frac{10x^2 + 10}{x-9} - 3\right] \left[\frac{x^2 - 18x - 1}{(x-9)^2}\right] \\ &= \frac{(10x^2 - 3x + 37)(x^2 - 18x - 1)}{(x-9)^3} \end{aligned}$$



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13.

$$\frac{dy}{du} = \frac{-3}{(2u-3)^2} \text{ et } \frac{du}{dx} = \frac{3x^2 - 32x - 45}{(3x-16)^2}$$

$$\frac{dy}{dx} = \frac{-3}{(2u-3)^2} \left[ \frac{3x^2 - 32x - 45}{(3x-16)^2} \right]$$

$$\frac{dy}{dx} = \frac{-3(3x^2 - 32x - 45)}{\left[ \frac{2x^2 + 6x - 2}{3x-16} - 3 \right]^2 (3x-16)^2}$$

$$\frac{dy}{dx} = \frac{-3(3x^2 - 32x - 45)}{(2x^2 - 3x + 46)^2}$$

14.  $-144(-12x^2 + 24x - 65)(x-1)$

15.  $\frac{3}{2}\sqrt{x}$

16.  $(10x^3 + 10x^2 - 70x + 37)(3x^2 + 2x - 7)$

17.  $\frac{3t^2 + 6}{2t + 1}$

18.  $10\sqrt{t} + 1$

19.  $-(20t^3 + 6t^2 + 1)t^2$

20.  $\frac{-5(t+1)^2}{(t-1)^2(t^2 + 2t - 1)}$

21.

$$\frac{dy}{dt} = 12t^3 - 2 \text{ et } \frac{dx}{dt} = 4 - \frac{1}{2\sqrt{t}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12t^3 - 2}{4 - \frac{1}{2\sqrt{t}}} = \frac{4\sqrt{t}(6t^3 - 1)}{8\sqrt{t} - 1}$$

22.

$$\frac{dy}{dt} = \frac{2t(t+4) - 1(t^2)}{(t+4)^2} = \frac{t^2 + 8t}{(t+4)^2}; \frac{dx}{dt} = \frac{13}{(t+4)^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t^2 + 8t}{(t+4)^2} \times \frac{(t+4)^2}{13} = \frac{t^2 + 8t}{13}$$

23.

$$\frac{dy}{ds} = 12s^2 + 6; \frac{dx}{ds} = 5$$

$$\frac{dy}{dx} = \frac{dy/ds}{dx/ds} = \frac{12s^2 + 6}{5}$$

24.

$$\frac{dy}{dr} = \frac{3}{4}r^{-1/4}; \frac{dx}{dr} = \frac{5}{(r+1)^2}$$

$$\frac{dy}{dx} = \frac{dy/dr}{dx/dr} = \left( \frac{3}{4}r^{-1/4} \right) \times \frac{(r+1)^2}{5} = \frac{3(r+1)^2}{20r^{1/4}}$$

25.

$$\frac{dy}{dt} = \frac{-4t^2 - 6t + 12}{(t^2 + 3)^2}; \frac{dx}{dt} = \frac{-1}{(t-7)^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left( \frac{-4t^2 - 6t + 12}{(t^2 + 3)^2} \right) \left( \frac{(t-7)^2}{-1} \right) = \frac{(4t^2 + 6t - 12)(t-7)^2}{(t^2 + 3)^2}$$

26.  $\frac{dy}{dx} = 4(x^2 + 5x + 6)^3(2x + 5)$

27.  $\frac{dy}{dx} = 7(5x^3 - 8x + 1)^6(15x^2 - 8)$

28.  $\frac{dy}{dx} = 8(7x - 15)^7(7)$

29.  $\frac{dy}{dx} = 5(x + 2\sqrt{x})^4 \left( 1 + \frac{1}{\sqrt{x}} \right)$

30.  $\frac{dy}{dx} = 3 \left( \frac{x-3}{2x+7} \right)^2 \left( \frac{1(2x+7) - 2(x-3)}{(2x+7)^2} \right) = \frac{39(x-3)^2}{(2x+7)^4}$

31.  $10(x^5 + x^4 - 2x + 7)^9(5x^4 + 4x^3 - 2)$

32.  $12(4x^3 - x^2 + 5x - 1)^{11}(12x^2 - 2x + 5)$

33.  $\frac{9}{2}\sqrt{(x^2 + 11x + 3)^7}(2x + 11)$

34.  $\frac{1}{2}\sqrt{\frac{x+3}{2x+17}} \left( \frac{-11}{(x+3)^2} \right)$

35.  $3 \left( \frac{x^2 + 5x - 1}{3x - 4} \right)^2 \left( \frac{3x^2 - 8x - 17}{(3x - 4)^2} \right)$

36.  $4 \left( \frac{x^3 - 3x + 2}{2x^2 + 5x} \right)^3 \left( \frac{2x^4 + 10x^3 + 6x^2 - 8x - 10}{(2x^2 + 5x)^2} \right)$

37.  $2(4x + 6)^3(2x - 5)^2(28x - 22) = 4(4x + 6)^3(2x - 5)^2(14x - 11)$

38.  $(x^2 + x + 2)(5x - 7)^2(35x^2 - 3x + 16)$

39.  $(3x^3 + 6x^2 - 5)^2(4x^2 + 5)^3(204x^4 + 336x^3 + 135x^2 + 20x)$

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40.

$$\begin{aligned} \frac{dy}{dx} &= 3(x^2 - 8)^2(2x)(2x^3 - 6x^2 + 7)^4 \\ &+ (x^2 - 8)^3 4(2x^3 - 6x^2 + 7)^3(6x^2 - 12x) \\ &= (x^2 - 8)^2(2x^3 - 6x^2 + 7)^3 [6x(2x^3 - 6x^2 + 7) \\ &+ (x^2 - 8)4(6x^2 - 12x)] \\ &= (x^2 - 8)^2(2x^3 - 6x^2 + 7)^3 6x [2x^3 - 6x^2 + 7 + 4x^3 \\ &- 8x^2 - 32x + 64] \\ &= (x^2 - 8)^2(2x^3 - 6x^2 + 7)^3(6x)(6x^3 - 14x^2 - 32x + 71) \end{aligned}$$

41.

$$\begin{aligned} \frac{dy}{dx} &= 2(3x^2 + 7)(6x)(x^8 - 2x^2 + 5)^3 \\ &+ (3x^2 + 7)^2 3(x^8 - 2x^2 + 5)^2(8x^7 - 4x) \\ &= (3x^2 + 7)(x^8 - 2x^2 + 5)^2 [12x(x^8 - 2x^2 + 5) \\ &+ (3x^2 + 7)3(8x^7 - 4x)] \\ &= (3x^2 + 7)(x^8 - 2x^2 + 5)^2(84x^9 + 168x^7 - 60x^3 - 24x) \end{aligned}$$

42.

$$\begin{aligned} \frac{dy}{dx} &= \frac{2x+1}{2\sqrt{x^2+x}}(2x+5)^3 + \sqrt{x^2+x} 3(2x+5)^2 \cdot 2 \\ &= \frac{(2x+1)(2x+5)^3 + 2(x^2+x)6(2x+5)^2}{2\sqrt{x^2+x}} \\ &= \frac{(2x+5)^2(16x^2+24x+5)}{2\sqrt{x^2+x}} \end{aligned}$$

43.

$$\begin{aligned} \frac{dy}{dx} &= 3(x+1)^2(2x+9)^2(3x-4)^4 \\ &+ (x+1)^3 2(2x+9)(2)(3x-4)^4 \\ &+ (x+1)^3(2x+9)^2 4(3x-4)^3(3) \\ &= (x+1)^2(2x+9)(3x-4)^3(54x^2 + 185x - 16) \end{aligned}$$

44.

$$\begin{aligned} \frac{dy}{dx} &= 2(x^3 + 4x)(3x^2 + 4)(5x - 3)^5(x^2 + 8)^3 \\ &+ (x^3 + 4x)^2 5(5x - 3)^4 5(x^2 + 8)^3 \\ &+ (x^3 + 4x)^2(5x - 3)^5 3(x^2 + 8)^2 2x \\ \frac{dy}{dx} &= (x^3 + 4x)(5x - 3)^4(x^2 + 8)^2(85x^5 - 36x^4 + 700x^3 \\ &- 240x^2 + 1120x - 192) \end{aligned}$$

54.

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{\sqrt[3]{x-1}}{2\sqrt{x+6}} - \frac{\sqrt{x+6}}{3\sqrt[3]{(x-1)^2}}}{(x-1)^{2/3}} = \frac{3(x-1) - 2(x+6)}{6(x+6)^{1/2}(x-1)^{4/3}} \\ &= \frac{x-15}{6\sqrt{x+6}\sqrt[3]{(x-1)^4}} \end{aligned}$$

45.

$$\begin{aligned} \frac{dy}{dx} &= \frac{-3}{2\sqrt{16-3x}}(4x-7)^2 + \sqrt{16-3x} 2(4x-7) 4 \\ \frac{dy}{dx} &= \frac{-3(4x-7)^2 + 2(16-3x)8(4x-7)}{2\sqrt{16-3x}} \\ \frac{dy}{dx} &= \frac{(4x-7)(-60x+277)}{2\sqrt{16-3x}} \end{aligned}$$

46.

$$\begin{aligned} \frac{dy}{dx} &= 3\left(\frac{x^2-6}{3x+4}\right)^2 \left(\frac{2x(3x+4) - 3(x^2-6)}{(3x+4)^2}\right) \\ \frac{dy}{dx} &= 3\left(\frac{x^2-6}{3x+4}\right)^2 \left(\frac{3x^2+8x+18}{(3x+4)^2}\right) \\ \frac{dy}{dx} &= \frac{3(x^2-6)^2(3x^2+8x+18)}{(3x+4)^4} \end{aligned}$$

47.

$$\begin{aligned} \frac{dy}{dx} &= \frac{3(x-5)^2(2x+7)^4 - 4(2x+7)^3(2)(x-5)^3}{(2x+7)^8} \\ \frac{dy}{dx} &= \frac{(x-5)^2(2x+7)^3(-2x+61)}{(2x+7)^8} = \frac{(x-5)^2(-2x+61)}{(2x+7)^5} \end{aligned}$$

$$48. \frac{(x^3-2x)(10x^3-6x^2+20x+4)}{(5x-1)^5}$$

49.

$$\begin{aligned} \frac{dy}{dx} &= \frac{9}{2\sqrt{9x-4}}(3x+8)^3 - 3(3x+8)^2 \cdot 3\sqrt{9x-4} \\ \frac{dy}{dx} &= \frac{9(3x+8)^3 - 9(3x+8)^2 \cdot 2(9x-4)}{2\sqrt{9x-4}(3x+8)^6} \\ \frac{dy}{dx} &= \frac{9(3x+8) - 18(9x-4)}{2\sqrt{9x-4}(3x+8)^4} = \frac{-135x+144}{2\sqrt{9x-4}(3x+8)^4} \end{aligned}$$

$$50. \frac{(4x-5)(12x+85)}{2(5+x)^{3/2}}$$

51.

$$\begin{aligned} \frac{dy}{dx} &= 3\left(\frac{x+3}{\sqrt{x}}\right)^2 \left(\frac{1\sqrt{x} - \frac{1}{2\sqrt{x}}(x+3)}{x}\right) \\ \frac{dy}{dx} &= 3\frac{(x+3)^2}{x} \times \frac{(x-3)}{2x\sqrt{x}} = \frac{3(x+3)^2(x-3)}{2x^2\sqrt{x}} \end{aligned}$$

$$52. \frac{dy}{dx} = 5\left(3x+1+\frac{1}{\sqrt{x}}\right)^4 \left(3-\frac{1}{2x\sqrt{x}}\right)$$

$$53. \frac{dy}{dx} = 3\left(x^2+\frac{2x-3}{x+1}\right)^2 \left(2x+\frac{5}{(x+1)^2}\right)$$

55.

$$\frac{dy}{dx} = \frac{\left[ \frac{3}{2\sqrt{3x+2}} (x^2 - 7) + \sqrt{3x+2} (2x) \right] (6x+5) - 6\sqrt{3x+2} (x^2 - 7)}{(6x+5)^2}$$

$$\frac{dy}{dx} = \frac{[3(x^2 - 7) + 4x(3x+2)](6x+5) - 12(3x+2)(x^2 - 7)}{2\sqrt{3x+2} (6x+5)^2}$$

$$\frac{dy}{dx} = \frac{54x^3 + 99x^2 + 166x + 63}{2\sqrt{3x+2} (6x+5)^2}$$

56.

$$\frac{dy}{dx} = \frac{\left[ \frac{3x-11}{2\sqrt{x+3}} + 3\sqrt{x+3} \right] \sqrt{2x+5} (4x-3) - \left[ \frac{4x-3}{\sqrt{2x+5}} + 4\sqrt{2x+5} \right] \sqrt{x+3} (3x-11)}{(2x+5)(4x-3)^2}$$

$$\frac{dy}{dx} = \frac{[3x-11+6(x+3)] \frac{\sqrt{2x+5} (4x-3)}{2\sqrt{x+3}} - [4x-3+4(2x+5)] \frac{\sqrt{x+3} (3x-11)}{\sqrt{2x+5}}}{(2x+5)(4x-3)^2}$$

$$\frac{dy}{dx} = \frac{(9x+7)(2x+5)(4x-3) - (12x+17) 2(x+3)(3x-11)}{2\sqrt{x+3} \sqrt{2x+5} (2x+5)(4x-3)^2}$$

57.

On a  $f(x) = 2x + 9$  et  $D_f = \mathbb{R}$

$$y = 2x + 9$$

$$\frac{y-9}{2} = x$$

$$f^{-1}(y) = \frac{y-9}{2}$$

$f^{-1}$  est une fonction.

De plus,  $D_{f^{-1}} = \mathbb{R} = I_f$

58.

$$f(x) = 3x^2 - 5 \quad D_f = \mathbb{R}$$

$$y = 3x^2 - 5$$

$$x = \pm \sqrt{\frac{y+5}{3}}$$

$$f^{-1}(y) = \pm \sqrt{\frac{y+5}{3}}$$

$f^{-1}$  n'est pas une fonction. De plus,

$$D_{f^{-1}} = \left\{ y \in \mathbb{R} \mid \frac{y+5}{3} \geq 0 \right\} = [-5, \infty[ = I_f$$

59.

$$f(x) = \frac{x+2}{x-3} \quad D_f = \mathbb{R} \setminus \{3\}$$

$$y = \frac{x+2}{x-3}$$

$$xy - 3y = x + 2$$

$$xy - x = 3y + 2$$

$$x = \frac{3y+2}{y-1}$$

$$f^{-1}(y) = \frac{3y+2}{y-1}$$

$f^{-1}$  est une fonction. De plus,

$$D_{f^{-1}} = \mathbb{R} \setminus \{1\} = I_f$$

## Calcul 42S : Exercices dérivation des fonctions composées et Implicite

60.  $f^{-1}(y) = (y+5)/8$  ; fonction;  $I_f = \mathbb{R}$

61.  $f^{-1}(y) = \sqrt[3]{y-1}$  ; fonction;  $I_f = \mathbb{R}$

62.  $f^{-1}(y) = \pm\sqrt{y^2+1}$  ; relation;  $I_f = \mathbb{R}^+ \cup \{0\}$

63.  $f^{-1}(y) = \frac{5y+5}{y-5}$  ; fonction;  $I_f = \mathbb{R} \setminus \{5\}$

64.

$$f(x) = 2 + \sqrt{x} \quad D_f = \{x \in \mathbb{R} \mid x \geq 0\} = \mathbb{R}^+ \cup \{0\}$$

$$y = 2 + \sqrt{x}$$

$$y - 2 = \sqrt{x}$$

$$x = (y - 2)^2$$

$$f^{-1}(y) = (y - 2)^2$$

$f^{-1}$  est une fonction. Cependant, comme  $x \geq 0$  alors  $y \geq 2$ .

$$D_{f^{-1}} = [2, \infty[ = I_f$$

65.

$$x = y^3 + 6y$$

$$\frac{dx}{dy} = 3y^2 + 6 \text{ donc } \frac{dy}{dx} = \frac{1}{3y^2 + 6}$$

66.

$$x = y + 3\sqrt{y}$$

$$\frac{dx}{dy} = 1 + \frac{3}{2\sqrt{y}} = \frac{2\sqrt{y} + 3}{2\sqrt{y}} ; \text{ donc } \frac{dy}{dx} = \frac{2\sqrt{y}}{2\sqrt{y} + 3}$$

67.

$$\frac{dx}{dy} = \frac{1(2y-1) - 2(y+2)}{(2y-1)^2} = \frac{-5}{(2y-1)^2}$$

$$\frac{dy}{dx} = \frac{(2y-1)^2}{-5}$$

68.

$$\frac{dx}{dy} = \frac{(2y+2)(y^3-y^2) - (3y^2-2y)(y^2+2y+6)}{(y^3-y^2)^2}$$

$$= \frac{-y^4 - 4y^3 - 16y^2 + 12y}{(y^3 - y^2)^2}$$

$$\frac{dy}{dx} = \frac{(y^3 - y^2)^2}{-y^4 - 4y^3 - 16y^2 + 12y}$$

69.  $\frac{1}{3y^2 + 10y}$

70.  $-(y+3)^2$

71.  $\frac{-(y^3+1)^2}{3y^2}$

72.  $\frac{(2y+3)^2}{4y^3+9y^2+6}$

73.  $\frac{\sqrt{y^2+2y+2}}{y+1}$

74.  $\frac{-xy}{x^2+1}$

76.

$$x^2y^2 + x^3y^3 - 4x + 8y = 0$$

$$2xy^2 + 2x^2yy' + 3x^2y^3 + 3x^3y^2y' - 4 + 8y' = 0$$

$$y' = \frac{4 - 2xy^2 - 3x^2y^3}{2x^2y + 3x^3y^2 + 8}$$

77.

$$2x + 5xy + 8x^2 - 16y^2 + 11 = 0$$

$$2 + 5y + 5xy' + 16x - 32yy' = 0$$

$$y' = \frac{-2 - 5y - 16x}{5x - 32y}$$

78.

$$x\sqrt{y} + y\sqrt{x} + xy = 3$$

$$\sqrt{y} + \frac{xy'}{2\sqrt{y}} + y'\sqrt{x} + \frac{y}{2\sqrt{x}} + y + xy' = 0$$

$$y' = \frac{-\sqrt{y} - y - \frac{y}{2\sqrt{x}}}{\frac{x}{2\sqrt{y}} + \sqrt{x} + x}$$

$$y' = \frac{(-2\sqrt{xy} - 2y\sqrt{x} - y)2\sqrt{y}}{2\sqrt{x}(x + 2\sqrt{xy} + 2x\sqrt{y})}$$

$$y' = \frac{-2\sqrt{xy^2} - 2y\sqrt{xy} - y\sqrt{y}}{x\sqrt{x} + 2\sqrt{x^2y} + 2x\sqrt{xy}}$$

79.

$$x^3y^2 + 5xy - 4x^2 + 6y = 11$$

$$3x^2y^2 + 2x^3yy' + 5y + 5xy' - 8x + 6y' = 0$$

$$y' = \frac{8x - 3x^2y^2 - 5y}{2x^3y + 5x + 6}$$

80.

$$xy + 3 = \frac{x + 2y^2}{xy - 4x}$$

$$x^2y^2 + 3xy - 4x^2y - 12x = x + 2y^2$$

$$2xy^2 + 2x^2yy' + 3y + 3xy' - 8xy - 4x^2y' - 12 = 1 + 4yy'$$

$$y' = \frac{13 - 2xy^2 - 3y + 8xy}{2x^2y + 3x - 4x^2 - 4y}$$

81.  $\frac{-y^3 - 18x^2}{3xy^2 + 3}$

82.  $\frac{-2xy - 2y^2}{4xy + x^2 - 6}$

83.  $\frac{-3x^2 - 5 - 3x^2y^3}{3x^3y^2}$

84.  $\frac{-y - 2xy^2 - 3x^2y^3}{x + 2x^2y + 3x^3y^2}$

85.  $\frac{-\sqrt{y}(14y\sqrt{x} + 1)}{\sqrt{x}(14x\sqrt{y} + 1)}$

86.  $\frac{-8y}{7x}$



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75.

$$xy^3 + 3y^2 - 6x = 0$$

$$y^3 + 3xy^2y' + 6yy' - 6 = 0$$

$$y' = \frac{6 - y^3}{3xy^2 + 6y}$$

87.  $\frac{y(1-x^2)}{x(1+x^2)} = \frac{16-2xy}{1+x^2}$

88.  $\frac{-11}{3}$

89.  $\frac{-1}{\sqrt{3}} = \frac{-1\sqrt{3}}{3}$

90.

a)  $[1, \infty[; \frac{1}{2\sqrt{x-1}}; ]1, \infty[$

b)  $\mathbb{R} \setminus \{4/3\}; \frac{-26}{(3x-4)^2}; \mathbb{R} \setminus \{4/3\}$

c)  $\mathbb{R}; \begin{cases} 1 & \text{si } x > -2 \\ -1 & \text{si } x < -2 \\ \cancel{\neq} & \text{si } x = -2 \end{cases}; \mathbb{R} \setminus \{-2\}$

91.

a) 29 400

b)  $\cancel{\neq}$

92.

a) 11016

b) 113,258

93.  $-9/5$