

1 Simplifier  $\left(\frac{a^{-1}b^{-2}}{a^2b^4}\right)^{-7} + \left(\frac{a^2b^{-3}}{a^{-1}b^2}\right)^{-7}$

2 Résoudre  $3^{2x+1} = 27$

3 Trouver la valeur de  $\log_3\left(\frac{1}{27}\right)$

4 Exprimer sous la forme d'un logarithme unique

$$\log 7 - 3\log 2 + \log 5 + \log 12 - \log 14$$

5 Trouver  $\log_7 31$ .

6 Résoudre  $\log_8\left(\frac{1}{16}\right) = x$

7 Résoudre  $(2,3)^x = 3,4$

8 Résoudre  $e^{2x} + 2e^x - 3 = 0$

9 Trouver le domaine et l'image de  $y = 5 + \log_5(2x+3)$

10 Montrer que  $\log_a uv = \log_a u + \log_a v$

Trouver la dérivée des fonctions suivantes :

11  $y = \ln(x+1)$

12  $y = e^{x+8}$

13  $y = \log_9(x^7 - x^6 + 3)$

14  $y = \ln\left(\frac{x-1}{x+7}\right)$

15  $y = x^2 \ln(x+1)$

16  $y = \sin x \times \ln(\sin x)$

17  $y = \log_5(x+3)^6$

18  $y = x \ln x + \sin x$

19  $y = e^x \sin x$

20  $y = \cos x \ln x$

21  $y = \frac{e^x}{\ln x}$

22  $y = 5^{x^2-3}$

23  $y = 4(3^{2x})$

24  $y = (3x+7)e^{x^2}$

25  $y = e^x(x+2 \ln x)$

26  $y = x^4 \ln x^2$

27  $y = \frac{e^x + 1}{\ln(x+1)}$

28  $y = \sin x \ln(2x - e^x)$

29  $y = e^x \arcsin 2x$

30  $y = \ln(\arctan x^2)$

31  $y = \cos e^{2x}$

Trouver  $dy/dx$  si

32  $y = 2^{x^2-x+1}$

33  $y = e^x \times \sin x \times \ln x$

48  $y = e^x \cos(x^2 + 1) \ln(x^3 + 2x) \arcsin x$

49  $y = \frac{(x+7)^3 e^{3x-1} \sin 3x}{x^2 \ln x}$

50  $y = \ln(\sin e^{x^2})$

51  $y = \ln \sqrt{x+5}$

52  $y = x^{x+1}$

53  $y = (\tan x)^{\sin x}$

54  $y = e^{x \tan x}$

55  $y = \frac{\tan x + e^x}{\ln x}$

56  $y = \frac{\sqrt[3]{x+1} \times \sin x \times e^{x^2} \times \ln(x^3 - 1)}{(2x+16)^2 \times \cos^3 x}$

57  $y = e^{-6x} \cos 3x^2$

58  $y = \ln\left(\frac{1}{1 + \sin^2 x}\right)$

59  $e^y = \sin(x \ln 3)$

60  $xy = e^{xy} - 2$

61  $y = \frac{(x^2 + 6) \times \sin x \times \arccos x^2}{(e^x + 2) \ln^3 x}$

62  $e^x \ln y = 3$

34  $y = e^{x^3}$

35  $y = e^{\ln x}$

36  $y = [\ln(x^3 + 1)]^2 \times e^{\sin x}$

37  $y = x^6 5^{x^3}$

38  $y = \frac{e^x}{e^x + 3}$

39  $y = \frac{\sin x}{e^x + \ln x}$

40  $y = \frac{1 + \tan x}{e^x}$

41  $xe^y = 2x + 4y$

42  $\ln xy = x + 3y - 4$

43  $e^x \ln y = 3 - e^{x^2} + y$

44  $y = (2x + 3)^{x+1}$

45  $y = (x^2 + 2)^{1+e^x}$

46  $y = (4x - 3)^{2^x}$

47  $e^x + e^y - x - y = 3$

63  $y = \frac{e^x \times (x^2 + 6x) \times \sec x}{\arctan x^2}$

64  $y = e^{2x} \times \ln(x+1) \times (x^3 + 3x + 7) \times \tan x \times \arcsin x$

65  $y = 2^x \times \log_7(x^2 + 1) \times \sec 3x \times \arccos x$

66 Trouver  $y^{(n)}$  si  $y = 7xe^x$

67 Trouver le plus petit nombre positif vérifiant l'équation  $4\cos x = e^x$

68 Trouver l'équation de la normale à la courbe  $y = 2x + \ln x$  au point  $(1, 2)$ .

Reponses

1.  $\frac{b^{16}}{a^3}$

2.  $3^{2x+1} = 3^5 \Rightarrow 2x+1=5 \Rightarrow x=2$

3.  $\log_3\left(\frac{1}{27}\right) = x \Rightarrow 3^x = \frac{1}{27} \Rightarrow 3^x = 3^{-3} \Rightarrow x = -3$

4.  $\log\left(\frac{15}{4}\right)$

5.  $\log_7 31 = \frac{\ln 31}{\ln 7} = \frac{3,4340}{1,9459} = 1,7647$

6.  $-\frac{4}{3}$

7. 1,4693

8.

$$e^{2x} + 2e^x - 3 = 0$$

$$(e^x - 1)(e^x + 3) = 0$$

$$e^x = 1 \Rightarrow x = 0$$

$$e^x = -3 \text{ pas de solution dans } \mathbb{R}$$

La seule solution est donc  $x = 0$ 

9.  $D = \left] -\frac{3}{2}, \infty \right[ ; I = \mathbb{R}$

10.

Soit  $x = \log_a u$  et  $y = \log_a v$  ;

alors,

$$a^x = u \text{ et } a^y = v$$

$$uv = a^x a^y = a^{x+y}$$

donc,

$$x + y = \log_a uv$$

c'est-à-dire,

$$\log_a u + \log_a v = \log_a uv$$

11.  $\frac{1}{x+1}$

12.  $e^{x+8}$

13.  $\frac{7x^6 - 6x^5}{x^7 - x^6 + 3} \log_a e$

14.  $\frac{8}{x^2 + 6x - 7}$

15.  $2x \ln(x+1) + \frac{x^2}{x+1}$

16.  $\cos x (1 + \ln \sin x)$

17.  $\frac{6 \log_3 e}{x+3}$

18.  $1 + \cos x + \ln x$

19.  $e^x (\sin x + \cos x)$

20.  $-\sin x \ln x + \frac{\cos x}{x}$

Calcul 42S  
Derive exponentielles et logarithmiques Exercice 14 D

$$21. e^x \left( \frac{\ln x - 1/x}{(\ln x)^2} \right)$$

$$22. \frac{dy}{dx} = 5^{x^2-3} \ln 5 (2x) = (2 \ln 5) x 5^{x^2-3}$$

$$23. \frac{dy}{dx} = 4 (3^{2x}) \ln 3 (2) = 8 \ln 3 (3^{2x})$$

$$24. \frac{dy}{dx} = 3 e^{x^2} + (3x+7) e^{x^2} 2x = e^{x^2} (6x^2 + 14x + 3)$$

$$25. \frac{dy}{dx} = e^x (x + 2 \ln x) + e^x (1 + 2/x) = e^x \left( x + 2 \ln x + 1 + \frac{2}{x} \right)$$

$$26. \frac{dy}{dx} = 4x^3 \ln x^2 + x^4 \left( \frac{1}{x^2} \right) 2x = 4x^3 \ln x^2 + 2x^3$$

$$27. \frac{dy}{dx} = \frac{e^x \ln(x+1) - \frac{1}{x+1} (e^x + 1)}{\ln^2(x+1)} = \frac{(x+1) e^x \ln(x+1) - (e^x + 1)}{(x+1) \ln^2(x+1)}$$

$$28. \frac{dy}{dx} = \cos x \ln(2x - e^x) + \sin x \left( \frac{2 - e^x}{2x - e^x} \right)$$

$$29. \frac{dy}{dx} = e^x \arcsin 2x + e^x \frac{2}{\sqrt{1-4x^2}}$$

$$30. \frac{dy}{dx} = \frac{1}{\arctan x^2} \times \frac{1}{1+x^4} \times 2x = \frac{2x}{(1+x^4) \arctan x^2}$$

$$31. \frac{dy}{dx} = (-\sin e^{2x}) e^{2x} 2 = -2e^{2x} \sin e^{2x}$$

45.

$$y = (x^2 + 2)^{1+e^x}$$

$$\ln y = (1 + e^x) \ln(x^2 + 2)$$

$$\frac{y'}{y} = e^x \ln(x^2 + 2) + \frac{(1 + e^x) 2x}{x^2 + 2}$$

$$y' = y \left[ e^x \ln(x^2 + 2) + \frac{2x(1 + e^x)}{x^2 + 2} \right]$$

46.

$$y = (4x - 3)^{2^x}$$

$$\ln y = 2^x \ln(4x - 3)$$

$$\frac{y'}{y} = 2^x \ln 2 \ln(4x - 3) + \frac{2^x (4)}{4x - 3}$$

$$y' = y 2^x \left[ \ln 2 \ln(4x - 3) + \frac{4}{4x - 3} \right]$$

47.

$$e^x + e^y - x - y = 3$$

$$e^x + y^y e^y - 1 - y^y = 0$$

$$y' = \frac{1 - e^x}{e^y - 1}$$

$$36. e^{\sin x} \ln(x^3 + 1) \left[ \frac{6x^2}{x^3 + 1} + \cos x \ln(x^3 + 1) \right]$$

$$37. 3x^5 5^{x^3} (2 + x^2 \ln 5)$$

$$38. \frac{3e^x}{(e^x + 3)^2}$$

$$39. \frac{xe^x(\cos x - \sin x) + x \cos x \ln x - \sin x}{x(e^x + \ln x)^2}$$

$$40. \frac{\tan^2 x - \tan x}{e^x}$$

41.

$$xe^y = 2x + 4y$$

$$e^y + xe^y y' = 2 + 4y'$$

$$y' = \frac{2 - e^y}{xe^y - 4}$$

42.

$$\ln xy = x + 3y - 4$$

$$\frac{y + xy'}{xy} = 1 + 3y'$$

$$y + xy' = xy + 3xyy'$$

$$y' = \frac{xy - y}{x - 3xy}$$

43.

$$e^x \ln y = 3 - e^{x^2} + y$$

$$e^x \ln y + e^x \frac{y'}{y} = -2xe^{x^2} + y'$$

$$ye^x \ln y + e^x y' = -2xy e^{x^2} + yy'$$

$$y' = \frac{-2xy e^{x^2} - y e^x \ln y}{e^x - y}$$

44.

$$y = (2x + 3)^{x+1}$$

$$\ln y = (x+1) \ln(2x+3)$$

$$\frac{y'}{y} = \ln(2x+3) + \frac{(x+1) 2}{(2x+3)}$$

$$y' = y \left[ \ln(2x+3) + \frac{2x+2}{2x+3} \right]$$

48.

$$y = e^x \cos(x^2 + 1) \ln(x^3 + 2x) \arcsin x$$

$$\ln y = \ln e^x + \ln \cos(x^2 + 1) + \ln \ln(x^3 + 2x) + \ln(\arcsin x)$$

$$\frac{y'}{y} = 1 + \frac{[-\sin(x^2 + 1)] 2x}{\cos(x^2 + 1)} + \frac{3x^2 + 2}{(x^3 + 2x) \ln(x^3 + 2x)} + \frac{1}{\sqrt{1-x^2} \arcsin x}$$

$$y' = y \left[ 1 - 2x \tan(x^2 + 1) + \frac{3x^2 + 2}{(x^3 + 2x) \ln(x^3 + 2x)} + \frac{1}{\sqrt{1-x^2} \arcsin x} \right]$$

49.

$$y = \frac{(x+7)^3 e^{3x-1} \sin 3x}{x^2 \ln x}$$

$$\ln y = \ln(x+7)^3 + \ln e^{3x-1} + \ln \sin 3x - \ln x^2 - \ln \ln x$$

$$\ln y = 3 \ln(x+7) + 3x - 1 + \ln \sin 3x - 2 \ln x - \ln \ln x$$

$$\frac{y'}{y} = \frac{3}{x+7} + 3 + \frac{3 \cos 3x}{\sin 3x} - \frac{2}{x} - \frac{1}{x \ln x}$$

$$y' = y \left[ \frac{3}{x+7} + 3 + 3 \cot 3x - \frac{2}{x} - \frac{1}{x \ln x} \right]$$

$$50. \frac{dy}{dx} = \frac{\cos e^{x^2}}{\sin e^{x^2}} \times e^{x^2} \times 2x = 2x e^{x^2} \cot e^{x^2}$$

$$51. \frac{1}{2(x+5)}$$

$$52. x^{x+1} [\ln x + (x+1)/x]$$

$$53. (\tan x)^{\sin x} (\cos x \ln \tan x + \sec x)$$

$$54. e^{x \tan x} (\tan x + x \sec^2 x)$$

$$55. \frac{x \sec^2 x \ln x + x e^x \ln x - e^x - \tan x}{x \ln^2 x}$$

56.

$$y' = y \left[ \frac{1}{3x+3} + \cot x + 2x + \frac{3x^2}{(x^3-1) \ln(x^3-1)} - \frac{2}{x+8} + 3 \tan x \right]$$

$$57. -6 e^{-6x} (\cos 3x^2 + x \sin 3x^2)$$

$$58. \frac{-\sin 2x}{1 + \sin^2 x}$$

$$59. \ln 3 \cot(x \ln 3)$$

$$60. -y/x$$

$$61. y' = y \left[ \frac{2x}{x^2+6} + \cot x - \frac{2x}{\sqrt{1-x^4} \arccos x^2} - \frac{e^x}{e^x+2} - \frac{3}{x \ln x} \right]$$

$$62. -y \ln y$$

$$63. y \left[ 1 + \frac{2x+6}{x^2+6x} + \tan x - \frac{2x}{(1+x^6) \arctan x^2} \right]$$

64.

$$y \left[ 2 + \frac{1}{(x+1) \ln(x+1)} + \frac{3x^2+3}{x^2+3x+7} + \frac{1}{\sin x \cos x} + \frac{1}{\sqrt{1-x^2} \arcsin x} \right]$$

$$65. y \left[ \ln 2 + \frac{2x \log_7 e}{(x^2+1) \log_7(x^2+1)} + 3 \tan 3x - \frac{1}{\sqrt{1-x^2} \arccos x} \right]$$

$$66. 7(x+n) e^x$$

$$67. 0,905$$

$$68. 3y + x - 7 = 0$$