Mathé 42S – Ex 14B

1. Trouve la dérivée des fonctions suivantes :

a) $y = \cos(3x+7)$

b) $y = \tan(7x+9)$

c) $y = x^2 - \cot x$

d) $\dfrac{\cos x}{x^2}$

e) $y = \sin(x^2+2)$

f) $y = \cot(x^4+2x)$

g) $y = \dfrac{2-\sin x}{2+\sin x}$

h) $y = \sec^3(\sqrt{x})$

i) $y = x^3 \cos\left(\dfrac{1}{x}\right)$

j) $y = \csc^2(2x)$

k) $y = \sqrt{\cot 3x}$

l) $y = \dfrac{\tan x}{x}$

m) $y = \sqrt{\sin x}$

n) $y = \dfrac{x^3}{\tan 3x}$

o) $y = x\sin(1-x^2)$

p) $y = x^3 \csc 6x$

q) $y = \sqrt{\csc 2x}$

r) $y = \sin^3 3x + \cos^2 2x$

s) $y = \csc(x^2+4)$

t) $y = \dfrac{\tan 2x}{1-\cot 2x}$

u) $y = \sin^4\left(\dfrac{\pi}{2}+x\right)$

v) $y = \sin^{11}(x-x^2)$

w) $y = x\sin x + \cos x$

x) $y = \dfrac{\sin x+\cos x}{\sin x-\cos x}$

y) $y = \sin(\cos(\tan x))$

z) $y = \sin\left(\dfrac{1}{\sqrt{x^2+1}}\right)$

2. $f(x) = \sqrt{1+\cos^2 x}$; trouve la valeur de $f'\left(\dfrac{\pi}{2}\right)$

3. $f(x) = \dfrac{\sin^2 x}{2+\sec^2 x}$; trouve la valeur de $f'\left(\dfrac{\pi}{3}\right)$

4. $f(x) = \dfrac{\sin 2x}{1-\cos x}$; trouve la valeur de $f'(\pi)$

5. Trouve la dérivée

a) $\sin x + \sin y = 1$ b) $\cos x + \sin y = 2$ c) $x\sin y + y\cos x = 3$

d) $5\sin x - 2\cos y = 4$ e) $x + y = x\sin(x + y)$ f) $x\tan y = y\tan x$

6. Trouve l'équation de la tangente et de la normale pour les courbes suivantes aux points indiqués.

a) $y = \cot x - \sqrt{2}\cos x$ au point $x = \dfrac{5\pi}{4}$

b) $y = \dfrac{1}{16}\sec^4 x$ au point $x = \dfrac{\pi}{3}$

Réponses:

a) $-3\sin(3x+7)$

b) $7\sec^2(7x+9)$

c) $2x + \csc^2 x$

d) $\dfrac{-x\sin x - 2\cos x}{x^3}$

e) $2x\cos(x^2+2)$

f) $-\csc^2(x^4+2x)(4x^3+2)$

g) $\dfrac{-\cos x(2+\sin x) - \cos x(2-\sin x)}{(2+\sin x)^2}$ ou $\dfrac{-4\cos x}{(2+\sin x)^2}$

h) $3\sec^2(\sqrt{x})\sec\sqrt{x}\tan\sqrt{x}\left(\dfrac{1}{2\sqrt{x}}\right)$

i) $3x^2\cos(1/x) + x\sin(1/x)$

j) $-4\csc^2 2x \cot 2x$

k) $\dfrac{-3\csc^2 3x}{2\sqrt{\cot 3x}}$

L) $\dfrac{x\sec^2 x - \tan x}{x^2}$

M) $\dfrac{\cos x}{2\sqrt{\sin x}}$

N) $\dfrac{3x^2 \tan 3x - 3x^3 \sec^2 3x}{\tan^2 3x}$

O) $\sin(1-x^2) - 2x^2\cos(1-x^2)$

P) $3x^2\csc 6x - 6x^3 \csc 6x \cot 6x$

Q) $-\dfrac{\csc 2x \cot 2x}{\sqrt{\csc 2x}}$

R) $9\sin^2 3x \cos 3x - 4\cos 2x \sin 2x$

S) $-2x\csc(x^2+4)\cot(x^2+4)$

T) $\dfrac{2\sec^2 2x(1-\cot 2x) - 2\csc^2 2x \tan 2x}{(1-\cot 2x)^2}$

U) $4\sin^3 x(\pi/2+x)\cos(\pi/2+x)$

V) $11\sin^{10}(x-x^2)\cos(x-x^2)(1-2x)$

w) $\sin x + x\cos x - \sin x = x\cos x$

x) $\dfrac{-2}{(\sin x - \cos x)^2}$

y) $\cos(\cos(\tan x))(-\sin(\tan x))\sec^2 x$

z) $\dfrac{-x}{(x^2+1)^{3/2}}\cos\left(\dfrac{1}{\sqrt{x^2+1}}\right)$

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2 a) $f'(x) = \dfrac{-\cos x \sin x}{\sqrt{1+\cos^2 x}}$ $\quad f'(\pi/2) = \dfrac{-(0)(1)}{\sqrt{1+0}} = 0$

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3 b) $f'(x) = \dfrac{(2\sin x\cos x)(2+\sec^2 x) - 2\sec^2 x \tan x \sin^2 x}{(2+\sec^2 x)^2}$ $\quad f'(\pi/3) = -\dfrac{\sqrt{3}}{12}$

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c)

4) $f'(x) = \dfrac{2\cos 2x(1-\cos x) + \sin x \cdot \sin 2x}{(1-\cos x)^2}$; $f'(\pi) = 1$

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5 a) $y' = \dfrac{-\cos x}{\cos y}$ $\quad$ b) $y' = \dfrac{\sin x}{\cos y}$ $\quad$ c) $y' = \dfrac{y\sin x - \sin y}{x\cos y + \cos x}$

d) $y' = \dfrac{-5\cos x}{2\sin y}$ $\quad$ e) $y' = \dfrac{x\cos(x+y) + \sin(x+y) - 1}{1 - x\cos(x+y)}$

f) $y' = \dfrac{y\sec^2 x - \tan y}{x\sec^2 y - \tan x}$

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6. a) $y = -3(x - 5\pi/4) + 2$ ← TANGENTE b) TANGENTE: $y = 4\sqrt{3}(x - \pi/3) + 1$

NORMALE: $y = \dfrac{1}{3}(x - 5\pi/4) + 2$ $\quad$ NORMALE: $y = \dfrac{-1}{4\sqrt{3}}(x - \pi/3) + 1$